CECS 228 Writing Assignment 4: Logical Reasoning

October 13th, 2021

Instructions

Submitting your work

Submit a single file with your solutions to the drop box by Thursday, October 14th, 8:00 pm. Make sure you provide your name and SID in the upper-right corner of your solution.

Late submissions

Should you submit after the dropbox deadline, solutions received no later than 30 minutes after the deadline will lose 20% of the earned points. Solutions received after 30 minutes but before midnight shall lose lose 50% of the earned points. All other late submissions will not be graded.

Problems

A. Consider the following propositional formulas that are assumed true. Using the inference rules and logic identities from the Reasoning lecture, provide a logical derivation that yields a contradiction. (10 pts)

1. $\neg c$
2. $d \rightarrow (\neg e \vee \neg a)$
3. $a \rightarrow (b \vee e)$
4. $\neg d \vee (b \rightarrow c)$

5. $\neg a \rightarrow c$
6. $\neg d \rightarrow (\neg a \vee \neg b)$
7. b
8. $\neg\neg a$ (1,5,Contrap.)
9. a (8, Double Neg.)
10. $a \wedge b$ (7,9 Conj.)
11. $\neg\neg(a \wedge b)$ (10, Double Neg.)
12. $\neg(\neg a \vee \neg b)$ (11, De Morgan)
13. $\neg\neg d$ (6,12,Contrap.)
14. $b \rightarrow c$, (4,13, OR)
15. c (7,14, Conditional)

Contradiction: 1,14

B. Now consider the formulas from problem A, but with $\neg b$ substituted for b. Derive a single conjunctive formula that establishes the truth (or falsehood) of Boolean variables a, b, d, and e. (10 pts)

1. $\neg c$
2. $d \rightarrow (\neg e \vee \neg a)$
3. $a \rightarrow (b \vee e)$
4. $\neg d \vee (b \rightarrow c)$
5. $\neg a \rightarrow c$
6. $\neg d \rightarrow (\neg a \vee \neg b)$
7. $\neg b$
8. $\neg\neg a$ (1,5,Contrap.)
9. a (8, Double Neg.)
10. $b \vee e$ (3,9, Conditional)
11. e (7,10,OR)
12. $e \wedge a$ (9,11 Conj.)
13. $\neg\neg(e \wedge a)$ (12, Double Neg.)
14. $\neg(\neg e \vee \neg a)$ (13, De Morgan)
15. $\neg d$ (2,14,Contrap.)
16. $a \wedge \neg b$ (7,9, Conj.)
17. $\neg d \wedge e$ (11,15, Conj.)
18. $a \wedge \neg b \wedge \neg d \wedge e$ (16,17, Conj.)