CECS 228 Group Assignment 2: Functions Puzzle

September 22, 2021

Instructions

Working together

Pair up with another student to solve both problems below. You are only allowed to discuss the problems with your partner and the lab instructor(s). Use of course notes, textbook, and lecture recordings is permitted, but no other online resources or outside communication is allowed. Each of you is responsible for authoring a (handwritten) solution to ONE of the problems. For example, if you author the solution to Problem A, then your partner authors the B solution.

Submitting your work

Submit ONE solution to the ONE problem you were assigned. Make sure you provide your name and SID as well as your partner's name in the upper-right corner of your solution, since both of you will receive points for each of the solutions. Upload your solution in a single file to the appropriate drop box before the end of class. Showing sufficient work is necessary for receiving maximum points.

Late submissions

Should you miss the dropbox deadline, email me your solution ASAP. Solutions received no later than 10 minutes after the deadline will lose 20% of the earned points. Solutions received after 10 minutes but less than 30 minutes after deadline will lose 50% of the earned points.
Bottom line: make it a goal to submit no later than 5 minutes before the drop-box dealine.

Functions Puzzle

Two functions $f : \{1, 2, 3, 4, 5\} \to \{a, b, c, d, e, h\}$ and $g : \{a, b, c, d, e, h\} \to \{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$ satisfy the following properties.

1. f is one-to-one while g is onto.
2. $f(5)$ and $g(e)$ begin with the same letter.
3. $f(\{2, 3\}) = \{a, e\}$
4. $f^{-1}(\{a, d, h\}) = \{3, 5\}$
5. $f^{-1}(\{b, e\}) = \{1, 2\}$
6. $g(\{a, c, h\}) = \{\diamondsuit, \heartsuit, \spadesuit\}$
7. $g(\{b, d\}) = \{\clubsuit, \diamondsuit\}$
8. $g^{-1}(\{\heartsuit, \spadesuit\}) = \{a, c\}$
9. $(g \circ f)(\{1, 3\}) = \{\clubsuit, \heartsuit\}$
10. $(g \circ f)^{-1}(\{\clubsuit, \spadesuit\}) = \{1, 4\}$

Problems

A. Solve the functions puzzle by determining f and g that satisfy each of the clues. Do this for each function by providing a table that shows the unique output assigned to each input. In other words, complete the following two tables.

Numerical Input n	**Letter Output** f(n)
1	
2	
3	
4	
5	

Letter Input l	**Suit Output** g(l)
a	
b	
c	
d	
e	
h	

Verify that each of the clues is satisfied. Show work. For example, when verifying that

$$(g \circ f)^{-1}(\{\clubsuit, \spadesuit\}) = \{1, 4\},$$

first compute $T = g^{-1}(\{\clubsuit, \spadesuit\})$, followed by $S = f^{-1}(T)$, and verify that $S = \{1, 4\}$. (15 points)

B. Consider the codomain value $f(4)$ that is part of your solution. For each of the other five codomain values, explain why $f(4)$ cannot equal that value. For example, if you determined that $f(4) = h$, then why is it not possible for $f(4) = a$? $f(4) = b$? etc.. Use complete sentences and separate paragraphs for each of the five codomain values that you analyze. (10 points)