CECS 228 Midterm, Part 1 of 3, Spring 2021, Dr. Ebert

A. In a ping pong tournament, each pair of players played each other in a match exactly once. Let variables x, y, z have domains equal to the set of players in the tournament. Moreover, predicate function $w(x, y)$ evaluates to `true` iff player x defeated player y in their match. Also, predicate function left(x) evaluates to `true` iff player x is lefthanded. Finally, predicate function champ(x) evaluates to `true` iff player x is the tournament champion. Use these variables and functions to translate the following statements into predicate logic.

1. The champion was not lefthanded. (7 pts)
2. Every lefthanded player won at least one match. (8 pts)
3. There was only one champion, and he won all his matches. (10 pts)

B. Each of the following predicate-logic formulas describes a real-valued function $f : R \to R$. For each one, provide a possible rule for $f(x)$. By "rule" we mean something like $f(x) = x^2$, $f(x) = \lfloor x \rfloor$, etc.. Assume all variables have domain equal to the set of real numbers.

1. $\exists y(y = \lfloor y \rfloor \wedge \forall x(f(x) = y^2))$ (7 pts)
2. $\forall x(f(x) > 0) \wedge \forall y \forall z(y < z \to f(y) < f(z))$ (8 pts)
3. $\forall y(y > 0 \to \exists x(x > 0 \wedge x \leq y \wedge f(x) \geq 1/y)$ (10 pts)

C. Consider the following propositional-logic formulas, where a, b, c, d, e are Boolean variables. Using the inference rules and identities from the Logical Reasoning lecture, and the assumptions given below, provide a logical derivation that establishes the truth values of each of the variables. To get you started, by assumption 5, we already know that c evaluates to `false`. Establish the truth values of the other four variables. Annotate all inferences. (25 pts)

1. $e \to (c \wedge \neg b)$
2. $c \oplus (a \to e)$
3. $\neg b \vee \neg d$
4. $\neg(a \wedge d) \to b$
5. $\neg c$