CECS 228 Midterm, Part 2 of 3, Fall 2021, Dr. Ebert

A. Suppose we want to prove that, if either integer a is even or integer b is even, then ab is even. If we use an indirect proof (i.e. contrapositive proof), what may we assume? What do we need to show? Hint: do NOT write the proof. (15 pts)

B. Consider the statement

$$\lceil x \rceil + \lceil y \rceil - \lceil x + y \rceil = 0 \text{ or } 1,$$

where x and y are assumed to be nonnegative real numbers. Suppose we write x and y respectively as $x = m + \epsilon$ and $y = n + \delta$, where m and n are nonnegative integers and $0 \leq \epsilon, \delta < 1$. Provide a set of cases for the values of ϵ and δ so that, for each case, the statement can be rewritten without the use of ceilings, and for which the right side of the equation is either 0 or 1. For each case, provide the resulting simplified statement. Hint: each simplified statement should use only m, n, and constants. (20 pts)

C. Consider the set of numbers $\{1, 2, \ldots, 12\}$. Suppose we partition this set into four disjoint subsets A, B, C, and D, where i) $|A| = |B| = |C| = |D| = 3$, and ii) no two sets overlap (e.g. $A \cap B = \emptyset$). Use a proof by contradiction to prove that the numbers in at least one of the four sets must sum to at least 20. For example, if we let $A = \{1, 6, 12\}$, $B = \{2, 5, 11\}$, $C = \{3, 4, 10\}$, and $D = \{7, 8, 9\}$, then these sets satisfy all the above conditions, and the numbers in D sum to $24 \geq 20$. Prove that this will always happen, no matter how we form the sets (so long as they satisfy the above conditions). (20 pts)

D. Let A denote the set of integers that are multiples of 6, B the set of integers that are multiples of 2, and C the set of integers that are multiples of 3. Use a set-containment proof to prove that $A = B \cap C$. Hint: your proof should make use of basic facts about integers, including prime numbers. (20 pts)