CECS 228 Midterm, Part 3 of 3, Fall 2021, Dr. Ebert

Please show all work and/or explain. Points will NOT be awarded for answers that have no justification. Please upload a single file to the drop box before the deadline.

A. Use mathematical induction to prove that

$$1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1,$$

for all $n \geq 0$.

1. Prove the basis step. (5 pts)
2. State the inductive assumption and what needs to be shown in the inductive step. (10 pts)
3. Based on your answer to 2, complete the inductive step. (10 pts)

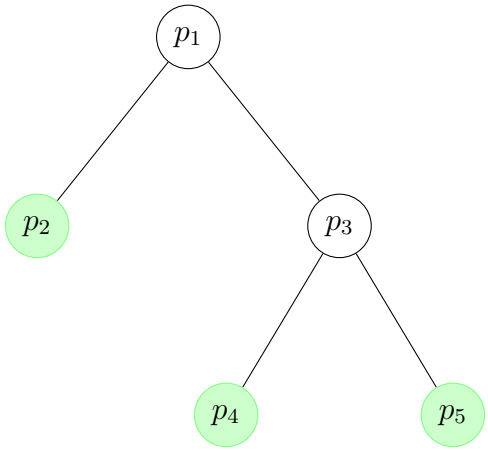
B. Use mathematical induction to prove that, if two bit strings x and y have the same length and numerical value, then it must be true that $x = y$, meaning that the bit sequence of x is the same bit sequence as y. For example, if $x = 0111$, x has numerical value $0 + 4 + 2 + 1 = 7$, and it is the only bit string of length four having this numerical value, meaning that, if y has numerical value 7, then necessarily $y = 0111$. Perform the induction on $n = |x| = |y| \geq 1$. Hint: make use Problem A above and use cases with respect to the most significant bits of x and y.

1. Prove the basis step. (5 pts)
2. State the inductive assumption and what needs to be shown in the inductive step. (10 pts)
3. Based on your answer to 2, complete the inductive step. (10 pts)

C. Pyramid Software company operates in the following way. When a programmer is a assigned a project having one task, the programmer completes the project/task. However, if the programmer is assigned a project having $n > 1$ tasks, the programmer divides the project into two subprojects, one having $\lfloor n/2 \rfloor$ tasks and the other having $\lceil n/2 \rceil$ tasks, hires two programmers, assigns each one a subproject, and supervises their work. For example, suppose programmer p_1 is assigned a project with three tasks. Then p_1 creates two subprojects, one with one task which is assigned to newly hired p_2, and one with two tasks that is assigned to newly hired p_3. Then p_2 completes his task, and p_3 divides his project into two subprojects, each having one task, and hires p_4 and p_5 who each complete one of the two tasks. Thus, 5 programmers (two supervisors and three coders) are needed to complete 3 programming tasks. The tree below provides the management hierarchy of the project. The leaves of the tree are the coders, and the higher-up nodes are supervisors. Use strong mathematical induction to prove that a project requiring $n \geq 1$ tasks will always require $2n - 1$ programmers (n coders and $n - 1$ supervisors).

1. Prove the basis step. (5 pts)

2. State the strong inductive assumption and what needs to be shown in the inductive step. (10 pts)
3. Based on your answer to 2, complete the inductive step. (10 pts)



Coders are colored green