Made with Goodnotes

Q3] a] $O(n^2)$ steps are just adding and subtracting a constant no. of $n/2 \times n/2$ matrices. $\therefore T(n) = 7T(n/2) + n^2$.

b] 5, −2, 0, 7, 3, 11, 2, 9, 5, 6

5, −2, 0, 7, 3   11, 2, 9, 5, 6

5, −2   0, 7   3   11, 2   9, 5   6

−2, 5   0, 7   3   2, 11   5, 9   6

−2, 5   0, 3, 7   2, 11   5, 6, 9

−2, 0, 3, 5, 7   2, 5, 6, 9, 11

−2, 0, 2, 3, 5, 6, 7, 9, 11

Made with Goodnotes

Q4] a] • For n even, w_n^j & $-w_n^j$ are both roots of unity. In other words, roots of unity come in additive pairs. If $0 \le j \le n/2$ then

$$w_n^{j+n/2} = -w_n^j$$

• Squares of n^{th} roots of unity yield the $n/2$ roots of unity

b] $DFT_4(5, 4, 3, -2)$

a_0: $DFT_2(5, 3)$

$(5, 5) + (3 \times 1, 3 \times -1)$

$= (8, 2)$

$DFT_1(5) = 5 \quad DFT_1(3) = 3$

a_1: $DFT_2(-4, -2)$

$(-4, -4) + (-2 \times 1, -2 \times -1)$

$= (-6, -2)$

$DFT_1(-6) = 6 \quad DFT_1(-2) = 2$

$\Rightarrow (8, 2, 8, 2) + (-6 \times 1, -2 \times i, -6 \times -1, -2 \times -i)$

$\Rightarrow (8, 2, 8, 2) + (-6, -2i, 6, 2i)$

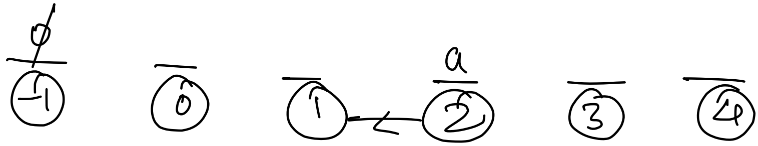
$\Rightarrow (2, 2-2i, 14, 2+2i)$

Made with Goodnotes

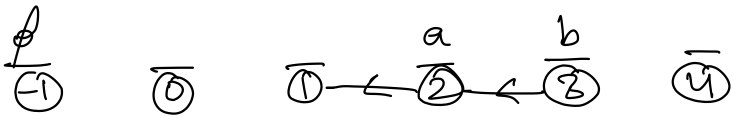
LO5) a] Refer to previous LO (11th October, 2023).

Made with Goodnotes

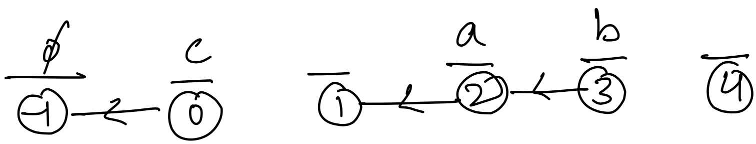
106] Insert (a, 2) p = 60



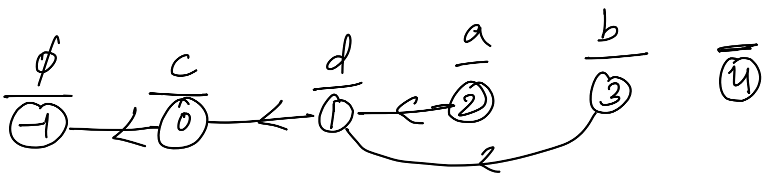
insert (b, 3) p = 50



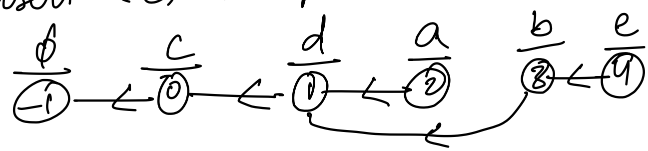
insert (c, 0) p = 40



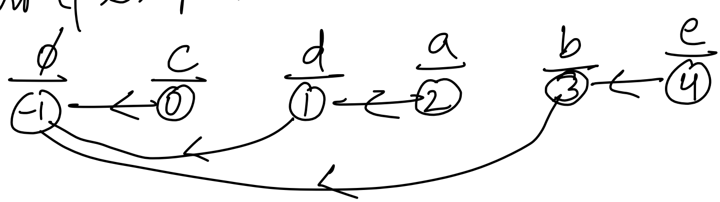
insert (d, 3) p = 30



insert (e, 4) p = 20



insert (f, 3) p = 10



Made with Goodnotes

Q7) a] mc(i,j) denotes the minimum multiplication complexity for the product $A_i, \ldots A_j$

b]

$$mc(i,j) = \begin{cases} 0 & \text{if } i = j \\ \min\limits_{i \le k < j} \left(mc(i,k) + mc(k+1,j) + p_{i-1} p_k p_j \right) & \text{if } i = j \end{cases}$$

c] $P_0 = 3 \quad P_1 = 4 \quad P_2 = 6 \quad P_3 = 1 \quad P_4 = 5$

i \ j	1	2	3	4
1	0	k=1 72	k=1 36	k=3 57
2	0	0	k=2 24	k=3 44
3	0	0	0	k=3 30
4	0	0	0	0

$mc(1,2) = (mc(1,1) + mc(2,2) + 3 \times 4 \times 6)$
$= \underline{72} \quad k=1$

$mc(2,3) = (mc(2,2) + mc(3,3) + 4 \times 6 \times 1)$
$= \underline{24} \quad k=2$

$mc(3,4) = (mc(3,3) + mc(4,4) + 6 \times 1 \times 5) = \underline{30} \quad k=3$

$MC(1,3) = \min(mc(1,1) + mc(2,3) + 3 \times 4 \times 1,\ mc(1,2) + mc(3,3) + 3 \times 6 \times 1)$
$= \min(0 + 24 + 12,\ 72 + 0 + 18) = \min(36, 90) = \underline{36 \quad k=1}$

$mc(2,4) = \min(mc(2,2) + mc(3,4) + 4 \times 6 \times 5,\ mc(2,3) + mc(4,4) + 4 \times 1 \times 5)$
$= \min(0 + 30 + 120,\ 24 + 0 + 20) = \min(150, 44) = \underline{44 \quad k=3}$

$MC(1,4) = \min(mc(1,1) + mc(2,4) + 3 \times 4 \times 5,\ mc(1,2) + mc(3,4) + 3 \times 6 \times 5,\ mc(1,3) + mc(4,4) + 3 \times 1 \times 5)$

$= \min(0 + 44 + 60,\ 72 + 30 + 90,\ 36 + 0 + 15)$

$= \min(104, 192, 57)$

$= 51 \quad k=3$

Made with Goodnotes