

Directions: show all work.

## Problems

LO1. Complete the following problems.

- (a) Use the Master Theorem to determine the growth of  $T(n)$  if it satisfies the recurrence  $T(n) = 49T(n/7) + n^2$ .
- (b) Use the substitution method to prove that, if  $T(n)$  satisfies

$$T(n) = 4T(n/2) + 5n,$$

Then  $T(n) = O(n^2)$ .

- a)  $n^{\log_7 49} = n^2 = f(n) \Rightarrow$  By Case 2 of the M.T. then  $T(n) = \Theta(n^2 \log n)$
- b) Inductive assumption: Assume  $T(k) \leq Ck^2 + dk$  for some constants  $C > 0$  and  $d$ .  
Show  $T(n) \leq cn^2 + dn$   
$$T(n) = 4T(n/2) + 5n \leq 4C\left(\frac{n}{2}\right)^2 + 4d\left(\frac{n}{2}\right) + 5n$$
$$= cn^2 + (\underline{d} + 5)n \leq cn^2 + dn \Leftrightarrow$$
$$dn \leq -5n, \Leftrightarrow \boxed{d \leq -5} \checkmark$$