# CECS 528, Learning Outcome Assessment 10, 12/4/2024, Dr. Ebert

LO6. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from problem $A$ to problem $B$.

(b) Given the integer function $f(n) = 3n^2 - 5n + 6$, is $f$ a valid reduction from `Even` to `Odd`? If no, then provide an example of why it's invalid. If yes, then establish its validity by stating and applying basic arithmetic properties of even and odd numbers.

LO7. An instance $(\mathcal{C}, m)$ of `Set Splitting` is a collection of subsets $\mathcal{C} = \{C_1, \ldots, C_n\}$, where, for each $i = 1, \ldots, n$, $C_i \subseteq \{1, 2, \ldots, m\}$. The problem is to decide if there is a set $A \subseteq \{1, 2, \ldots, m\}$ for which, for every $i = 1, \ldots, n$, both $A \cap C_i \neq \emptyset$ and $\overline{A} \cap C_i \neq \emptyset$, where $\overline{A} = \{1, 2, \ldots, m\} - A$. Do the following to establish that `Set Splitting` is a member of `NP`.

(a) For a given instance $(\mathcal{C}, m)$ of `Set Splitting` describe a certificate in relation to $(\mathcal{C}, m)$.

$A$ is a subset of $\{1, 2, \ldots, m\}$

(b) Provide a semi-formal verifier algorithm that takes as input i) an instance $(\mathcal{C}, m)$ of `Set Splitting`, ii) a certificate for $(\mathcal{C}, m)$ as defined in part a, and decides if the certificate is valid for $(\mathcal{C}, m)$.

For each $i = 1, \ldots, n$,
  If $A \cap C_i = \emptyset \vee \overline{A} \cap C_i = \emptyset$
    return 0.
Return 1.

(c) Provide size parameters that may be used to measure the size of an instance $(\mathcal{C}, m)$ of `Set Splitting`.

$n = |\mathcal{C}|$  $m =$ bound on the size of any $C_i \in \mathcal{C}$.

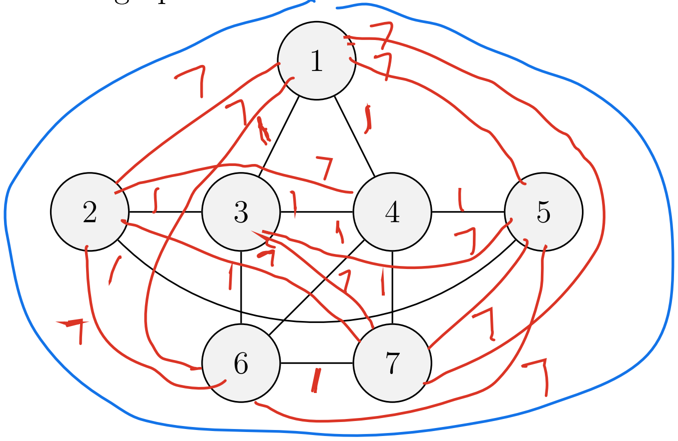
(d) Use the size parameters from part c to describe the running time of your verifier from part b. Defend your answer in relation to the algorithm you provided for the verifier.

Assuming that each $C_i$ is represented as a sorted array, as well as $A$ and $\overline{A}$, each iteration of the for-loop can be executed in $O(m)$ steps.
$\therefore$ the verifier requires $O(mn)$ steps which is polynomial with

respect to the size parameters.

LO8. The graph below is an instance of `Hamilton Cycle (HC)`

$f(G) =$



$k = 7$

(a) Draw $f(G)$, where $f$ : `HC` $\to$ `TSP` is the mapping reduction from `HP` to `Traveling Salesperson (TSP)` described in lecture.

(b) Verify that $f$ is valid for input $G$ from part a in the sense that both $G$ and $f(G)$ are either both positive instances or both negative instances. Make sure your answer is specific to the provide graph. Defend your answer.

G is a negative instance of HC since if starting from 2 and when moving to 1, say from 3, ~~the next vertex~~ would have to be 4 which does not allow for a way to return to 2, since 5 would have to be visited next, but then there is no way to reach 6 and 7. Thus, an HC in f(G) would have to make use of a red edge, and its cost would exceed 7.

LO9. Do the following.

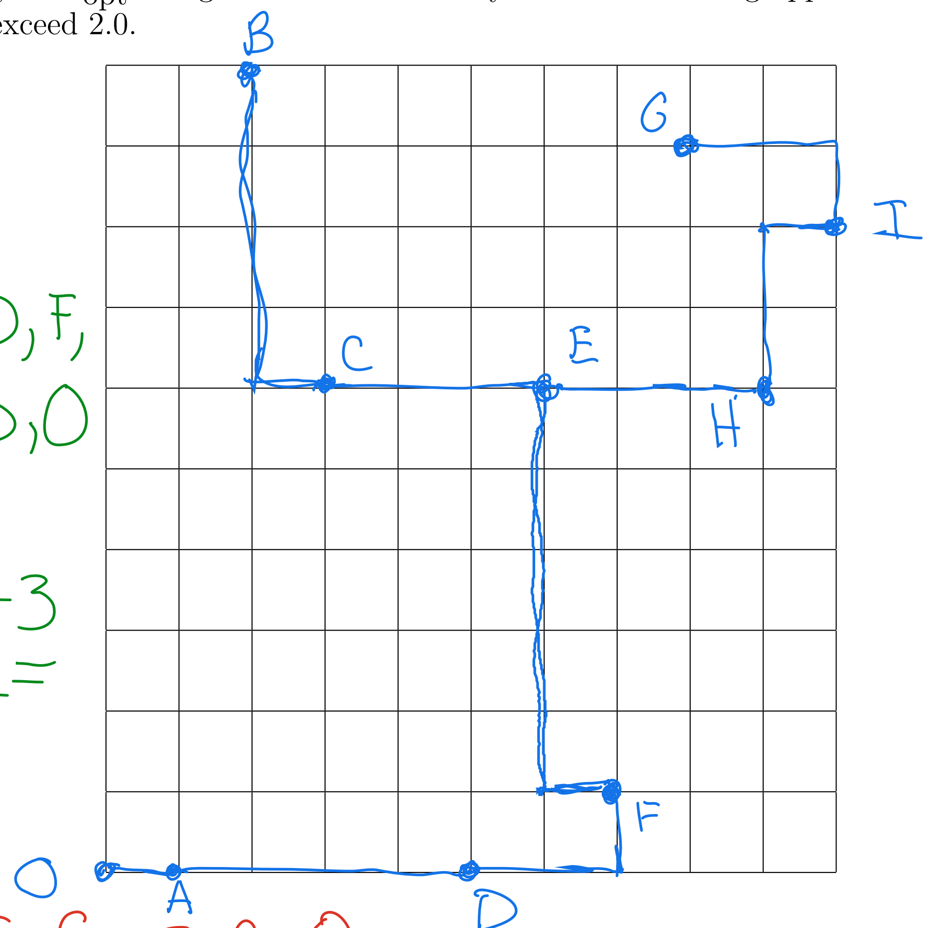
(a) A delivery truck starts at location $O = (0, 0)$, and must deliver packages to locations

$$A = (1, 0), B = (2, 10), C = (3, 6), D = (5, 0), E = (6, 6), F = (7, 1), G = (8, 9), H = (9, 6),$$

$$\text{and } I = (9, 8)$$

before returning back to $O$. The distance from one location to another is measured by adding both the number of horizontal and vertical units that separate them. For example, location $D$ is $1 + 6 = 7$ units from location $E$. Provide a tour for the driver, using the 2-approximation algorithm for TSP provided in lecture. Plot all locations (including $O$) using the provided grid and draw the mst $T$ that connects them. Assuming that $T$ is rooted at location $O$, provide the resulting approximation delivery cycle $C$ along with its cost. Finally, based on visual inspection, provide what you believe to be the optimal delivery cycle $C_{\text{opt}}$ along with its cost. Verify that the resulting approximation ratio is does not exceed 2.0.

MST $T$ — Depth-First Traversal:




$C_{approx} =$ O, A, D, F, E, H, I, G, C, B, O

$Cost(C_{approx}) = 1 + 4 + 3 + 6 + 3 + 3 + 3 + 8 + 5 + 12 = 48$

$C_{opt} =$ O, A, D, F, E, H, I, G, B, C, O

$Cost(C_{opt}) = 1 + 4 + 3 + 6 + 3 + 3 + 3 + 7 + 5 + 9 = 44$

Approx Ratio $= \frac{48}{44} = \frac{12}{11} < 2.0$.

(b) Explain the reasoning behind why the 2-approximation algorithm for TSP always produces an approximation cycle whose cost is at most twice the cost of the optimal solution.

LO10. Do the following.

(a) Given random variable $X$ with $\text{dom}(X) = \{1, 2, 3\}$, $p_1 = 2/3$, $p_2 = 1/6$, and $p_3 = 1/6$, compute $E[X]$.

$$E[X] = (1)\left(\frac{2}{3}\right) + (2)\left(\frac{1}{6}\right) + (3)\left(\frac{1}{6}\right) =$$

$$\frac{2}{3} + \frac{1}{3} + \frac{1}{2} = 1.5$$

(b) Consider the graph $G$ from the LO6 problem on page 1 of this assessment. Suppose we wish to randomly select one of its vertices, where the likelihood of selecting a vertex is directly proportional to its degree. Define a random variable $V$ which meets these requirements. Your answer should be specific to $G$.

$$\text{dom}(V) = \{1, 2, \ldots, 7\}$$

$$(p_1, \ldots, p_7) = \left(\frac{2}{20}, \frac{2}{20}, \frac{4}{20}, \frac{5}{20}, \frac{2}{20}, \frac{3}{20}, \frac{2}{20}\right) =$$

$$= (0.1, 0.1, 0.2, 0.25, 0.1, 0.15, 0.1)$$

Note: $\sum_{i=1}^{7} p_i = 1.$