CECS 528, Learning Outcome Assessment 10, 12/4/2024, Dr. Ebert

LO6. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from problem A to problem B.

(b) Given the integer function $f(n) = 3n^2 - 5n + 6$, is f a valid reduction from `Even` to `Odd`? If no, then provide an example of why it's invalid. If yes, then establish its validity by stating and applying basic arithmetic properties of even and odd numbers.

LO7. An instance $(\mathcal{C}, m)$ of `Set Splitting` is a collection of subsets $\mathcal{C} = \{C_1, \ldots, C_n\}$, where, for each $i = 1, \ldots, n$, $C_i \subseteq \{1, 2, \ldots, m\}$. The problem is to decide if there is a set $A \subseteq \{1, 2, \ldots, m\}$ for which, for every $i = 1, \ldots, n$, both $A \cap C_i \neq \emptyset$ and $\overline{A} \cap C_i \neq \emptyset$, where $\overline{A} = \{1, 2, \ldots, m\} - A$. Do the following to establish that `Set Splitting` is a member of `NP`.

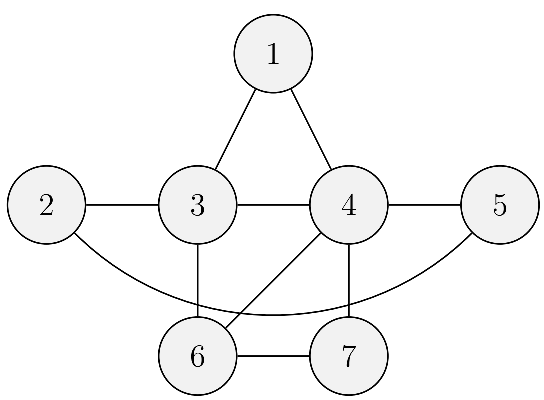
(a) For a given instance $(\mathcal{C}, m)$ of `Set Splitting` describe a certificate in relation to $(\mathcal{C}, m)$.

(b) Provide a semi-formal verifier algorithm that takes as input i) an instance $(\mathcal{C}, m)$ of `Set Splitting`, ii) a certificate for $(\mathcal{C}, m)$ as defined in part a, and decides if the certificate is valid for $(\mathcal{C}, m)$.

(c) Provide size parameters that may be used to measure the size of an instance $(\mathcal{C}, m)$ of `Set Splitting`.

(d) Use the size parameters from part c to describe the running time of your verifier from part b. Defend your answer in relation to the algorithm you provided for the verifier.

LO8. The graph below is an instance of `Hamilton Cycle (HC)`



(a) Draw $f(G)$, where f : `HC` $\rightarrow$ `TSP` is the mapping reduction from `HP` to `Traveling Salesperson (TSP)` described in lecture.

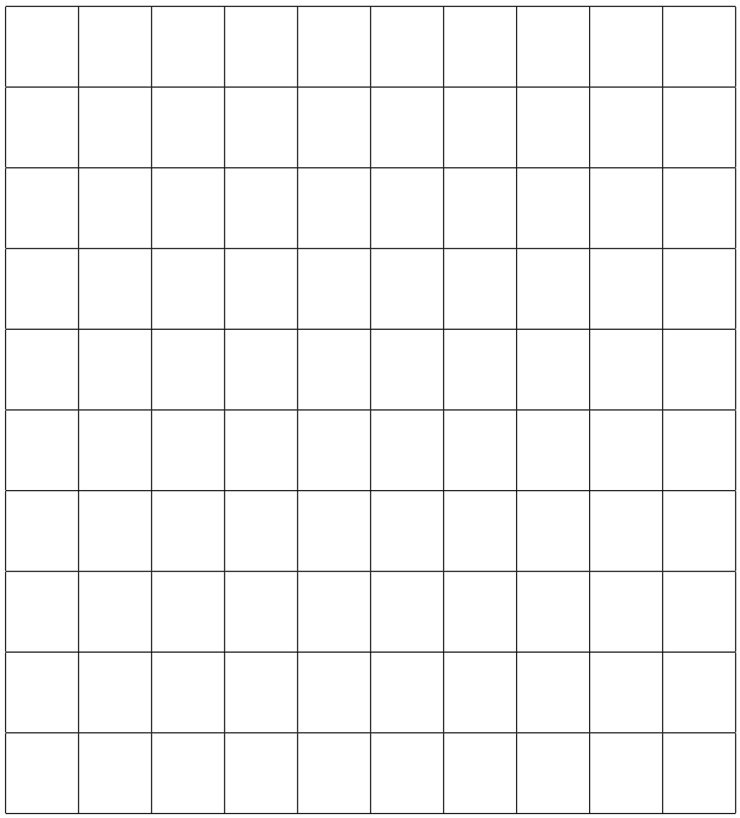
(b) Verify that f is valid for input G from part a in the sense that both G and $f(G)$ are either both positive instances or both negative instances. Make sure your answer is specific to the provide graph. Defend your answer.

LO9. Do the following.

(a) A delivery truck starts at location $O = (0, 0)$, and must deliver packages to locations

$$A = (1, 0), B = (2, 10), C = (3, 6), D = (5, 0), E = (6, 6), F = (7, 1), G = (8, 9), H = (9, 6),$$
$$\text{and } I = (9, 8)$$

before returning back to O. The distance from one location to another is measured by adding both the number of horizontal and vertical units that separate them. For example, location D is $1 + 6 = 7$ units from location E. Provide a tour for the driver, using the 2-approximation algorithm for TSP provided in lecture. Plot all locations (including O) using the provided grid and draw the mst T that connects them. Assuming that T is rooted at location O, provide the resulting approximation delivery cycle C along with its cost. Finally, based on visual inspection, provide what you believe to be the optimal delivery cycle C_{opt} along with its cost. Verify that the resulting approximation ratio is does not exceed 2.0.



(b) Explain the reasoning behind why the 2-approximation algorithm for TSP always produces an approximation cycle whose cost is at most twice the cost of the optimal solution.

LO10. Do the following.

(a) Given random variable X with $\text{dom}(X) = \{1, 2, 3\}$, $p_1 = 2/3$, $p_2 = 1/6$, and $p_3 = 1/6$, compute $E[X]$.

(b) Consider the graph G from the LO6 problem on page 1 of this assessment. Suppose we wish to randomly select one of its vertices, where the likelihood of selecting a vertex is directly proportional to its degree. Define a random variable V which meets these requirements. Your answer should be specific to G.