CECS 528, LO12 Assessment Problem, Fall 2024, Dr. Ebert

Directions: Please review the `Homework` section on page 6 of the syllabus including a list of all rules and guidelines for writing and submitting solutions.

Due Date: Monday, December 16th at the beginning of class.

Problem

LO12. Consider the problem of applying `WalkSAT` to a satisfiable instance of `3SAT` that has four variables and a unique satisfying assignment β. Letting F be the random variable that measures the number of bit flips leading up to the event $\alpha = \beta$, our goal is to compute $E[F|D = i]$, $i = 0, 1, \ldots, 4$, where D is a random variable that represents the current Hamming distance $d(\alpha, \beta)$.

(a) Determine the values $E[F|D = 0], \ldots, E[F|D = 4]$, using both recurrences and edge cases that are inspired by the recurrence equation provided in item 6 of the proof outline of Theorem 5.1 of the Randomized Algorithms lecture. As an example, for $i = 3$, we have

$$E[F|D = 3] = p(1 + E[F|D = 2]) + (1 - p)(1 + E[F|D = 4]).$$

Here p is chosen by considering the following worst-case scneario. Suppose c is a randomly selected unsatisfied clause. Since $D = 3$, at most one of the literals of c is correctly assigned. Thus if a literal of c is randomly and uniformly selected, then there is a probability of $p = 2/3$ that flipping the assignment bit for this literal will result in one additonal literal being in agreement with β, i.e. reducing the Hamming distance from 3 to 2. Use similar analyses for the cases $D = 1$ and $D = 2$, along with the two edge cases $D = 0$ and $D = 4$ to obtain five equations with respect to five variables and solve the system of equations. (12 pts)

(b) Use the results of part a along with the expectation of conditional expectation to compute $E[F]$, the expected number of bit flips needed by `WalkSAT` to obtain $\alpha = \beta$. Hint: D is binomially distributed. (8 pts)

(c) Simulate the `WalkSAT` algorithm 5 times using `3SAT` instance

$$\mathcal{C} = \{(x_1, x_2, x_3), (\overline{x}_2, x_3, \overline{x}_4), (x_1, x_2, \overline{x}_4), (\overline{x}_1, \overline{x}_3, \overline{x}_4), (\overline{x}_1, x_2, x_4), (\overline{x}_2, x_3, x_4), (x_1, \overline{x}_3, x_4),$$

$$(\overline{x}_2, \overline{x}_3, x_4), (\overline{x}_2, \overline{x}_3, \overline{x}_4)\},$$

where simulation continues until satisfying assignment $\beta = (x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 1)$ has been reached. Compare the average of these empirical F-values with your answer from part b. For each simulation, provide a table where each row indicates the unsatisfied clauses, the random clause selected, the random literal selected, and the updated α assignment. (5 pts)