CECS 528, Learning Outcome Assessment 2, 9/11/2024, Dr. Ebert

Directions: show all work.

Problem

LO1. Solve the following problems.

(a) Use the Master Theorem to determine the growth of $T(n)$ if it satisfies the recurrence $T(n) = 5T(n/2) + n^{\log_3 27}$. Defend your answer.

(b) Use the substitution method to prove that, if $T(n)$ satisfies

$$T(n) = T(n/3) + T(2n/3) + n$$

then $T(n) = \Omega(n \log n)$.

LO2. Solve each of the following problems.

(a) Consider the following algorithm of Karatsuba called `multiply` for multiplying two n-bit binary numbers x and y. In what follows, we assume n is even. Let x_L and x_R be the leftmost $n/2$ and rightmost $n/2$ bits of x respectively. Define y_L and y_R similarly. Let P_1 be the result of calling `multiply` on inputs x_L and y_L, P_2 be the result of calling `multiply` on inputs x_R and y_R, and P_3 the result of calling `multiply` on inputs $x_L + x_R$ and $y_L + y_R$. Then return the value $P_1 \times 2^n + (P_3 - P_1 - P_2) \times 2^{n/2} + P_2$. Provide a divide-and-conquer recurrence for the running time of this algorithm. Justify your choices of a, b, and $f(n)$.

(b) Recall that the `Minimum Positive Subsequence Sum (MPSS)` problem admits a divide-and-conquer algorithm that, on input integer array a, requires computing the mpss of any subarray of a that contains both $a[n/2-1]$ and $a[n/2]$ (the end of a_{left} and the beginning of a_{right}). For

$$a = 33, -34, -1, -31, 14, -20, 61, -7, 83, -10$$

provide the two sorted arrays $a = \text{SortedLeftSums}$ and $b = \text{SortedRightSums}$ from which the minimum positive sum $a[i] + b[j]$ represents the desired mpss (for the middle), where i in the index range of a and j is within the index range of b. Also, demonstrate how the minimum positive sum $a[i] + b[j]$ may be computed via the movement of left and right markers.