

Directions: show all work.

## Problem

LO1. Solve the following problems.

- (a) Use the Master Theorem to determine the growth of  $T(n)$  if it satisfies the recurrence  $T(n) = 7T(n/2) + n^2$ . Defend your answer.
- (b) Use the substitution method to prove that, if  $T(n)$  satisfies

$$T(n) = 2T(n/2) + n^2$$

then  $T(n) = \Omega(n^2)$ .

LO2. Solve each of the following problems.

- (a) Recall that the **Find-Statistic** algorithm makes use of the Partitioning algorithm and uses a pivot that is guaranteed to have at least

$$3(\lfloor \frac{1}{2} \lceil \frac{n}{5} \rceil \rfloor - 2) \geq 3(\frac{1}{2} \cdot \frac{n}{5} - 3) = \frac{3n}{10} - 9.$$

members of array  $a$  both to its left and to its right. Rewrite each of the above inequalities but now assuming the algorithm uses groups of nine instead of groups of five. Explain your reasoning for each of the numerical changes that you make.

- (b) Recall that the **Maximum Subsequence Sum (MSS)** problem admits a divide-and-conquer algorithm that, on input integer array  $a$ , requires computing the mss of any subarray of  $a$  that contains both  $a[n/2 - 1]$  and  $a[n/2]$  (the end of  $a_{\text{left}}$  and the beginning of  $a_{\text{right}}$ ). For

$$a = -2, 6, -1, 3, -4, 4, -5, 4, -1, 3$$

provide the two arrays  $S_{\text{LeftSums}}$  and  $S_{\text{RightSums}}$  from which the mss (for the middle) can be computed. Based on these two arrays what is the largest mss that includes members of both  $a_{\text{left}}$  and  $a_{\text{right}}$ ? In general, how many steps does it take to compute this mss, including computing  $S_{\text{LeftSums}}$  and  $S_{\text{RightSums}}$ ? Explain.

LO3. Solve each of the following problems.

- (a) For the weighted graph with edges

$$(a, e, 6), (b, e, 4), (c, e, 3), (c, d, 5), (d, f, 2), (e, f, 1),$$

Show how the disjoint-set data structure forest changes when processing each edge in Kruskal's sorted list of edges. When unioning two trees, use the convention that the root of the union is the root which has the *lower* alphabetical order. For example, if two trees, one with root  $a$ , the other with root  $b$ , are to be unioned, then the unioned tree should have root  $a$ .

- (b) The **Fuel Reloading Problem** is the problem of traveling in a vehicle from one point to another, with the goal of minimizing the number of times needed to re-fuel. It is assumed that travel starts at point 0 (the origin) of a number line, and proceeds right to some final integer point  $F > 0$ . The input includes  $F$ , a list of stations  $0 < s_1 < s_2 < \cdots < s_n < F$ , and a distance  $d$  that the vehicle can travel on a full tank of fuel before having to re-fuel. Describe the greedy choice the driver can make in order to minimize the number times that the vehicle has to be re-fueled. Using your answer as the basis for a greedy algorithm, for the given instance  $d = 7$ ,  $F = 50$ , and station locations

$$\{2, 4, 10, 13, 17, 19, 25, 26, 30, 37, 43, 46\},$$

determine the stations that the greedy driver would use to re-fuel?