CECS 528, Learning Outcome Assessment 7, 10/23/2024, Dr. Ebert

Problem

LO3. Solve each of the following problems.

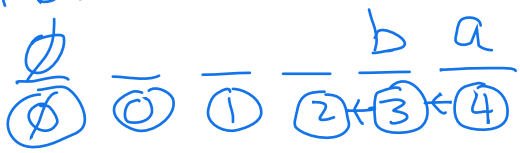
(a) Recall the use of the disjoint-set data structure for the purpose of improving the running time of the `Unit Task Scheduling (UTS)` algorithm. For the set of tasks

Task	a	b	c	d	e	f
Deadline	4	3	4	2	4	4
Profit	60	50	40	30	20	10

show the M-Tree forest after it has been inserted (or at least has attempted to be inserted in case the scheduling array is full). Notice that the earliest deadline is 0, meaning that the earliest slot in the schedule array has index 0. Hint: to receivec credit, your solution should show six different snapshots of the M-Tree forest.

Insert a:

Insert b:



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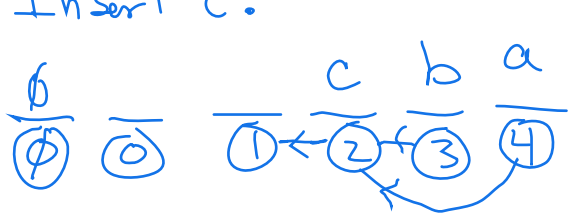

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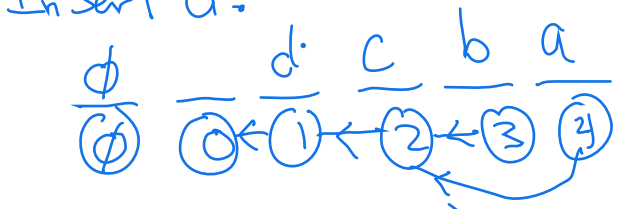
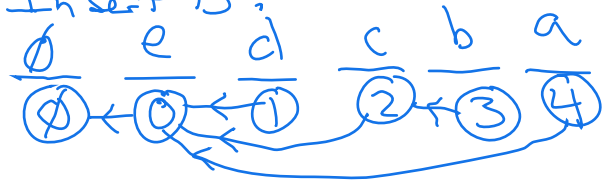
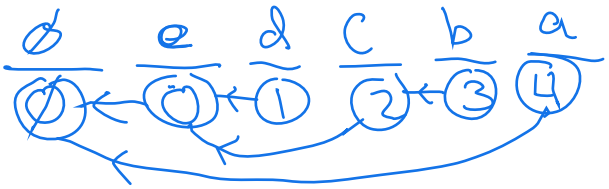


Insert b:



Insert f (fail):



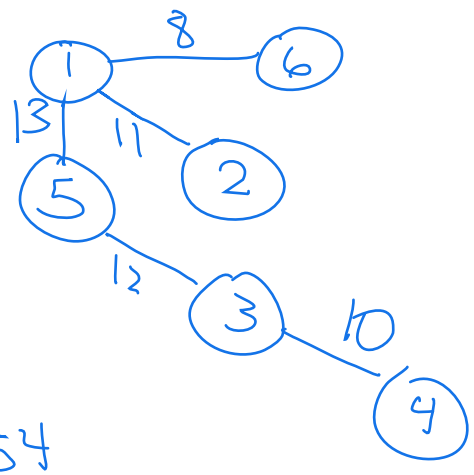
(b) State the greedy choice that is being made in each round of Prim's algorithm. The weighted edges of a graph $G = (V, E)$ are

$$E = \{(1,2,11), (1,3,19), (1,4,15), (1,5,13), (1,6,8), (2,3,14), (2,4,17),$$

$$(2,5,16), (2,6,22), (3,4,10), (3,5,12), (3,6,19), (4,5,15), (5,6,20)\}.$$

For each round of Prim's algorithm applied to G, indicate the selection for that round and provide a drawing of the final output of the algorithm. Hint: you do *not* need to use a heap data structure. Also, break any ties by choosing the vertex having least value. For example, if there is a tie beetween vertices 2 and 4, then choose vertex 2.

Round	Selected edge / Vertex
1	n/a / 1
2	(1,6) / 6
3	(1,2) / 2
4	(1,5) / 5
5	(3,5) / 12
6	(3,4) / 10



Cost = 54

LO4. Answer the following.

(a) The dynamic-programming algorithm that solves the `Optimal Binary Search Tree` optimization problem defines a recurrence for the function wac(i, j). In words, what does wac(i, j) equal? Hint: do *not* write the recurrence (see Part b).

See Notes

(b) Provide the dynamic-programming recurrence for wac(i, j).

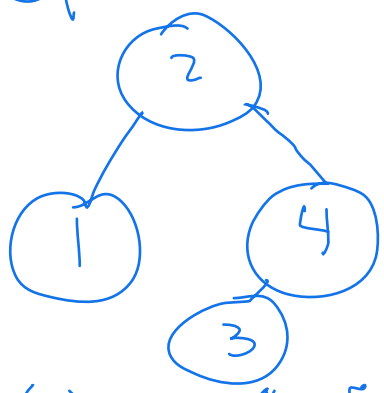
See Notes

(c) Apply the recurrence from Part b to an instance having keys 1-4, and respective weights 60,40,30,50. Record all subproblem solutions (including the k-values) in a matrix and use the matrix to draw an optimal solution.

Wac(i,j)

i \ j	1	2	3	4
1	60	140 k=1	220 k=2	350 k=2
2	0	40	100 k=2	210 k=3
3	0	0	30	110 k=4
4	0	0	0	50

Optimal Tree T:



Wac(T) = 40 + (60+50)(2)+(3)(30) = 350

Wac(1,3) = min(0+Wac(2,3), 60+30, W(1,2)+0) +130

min(100, 90, 140) + 130 = 220 k=2

Wac(2,4) = min(0 + wac(3,4), 40+50, Wac(2,3)+0) + 120

= min(110, 90, 100) + 120 = 210 at k=3

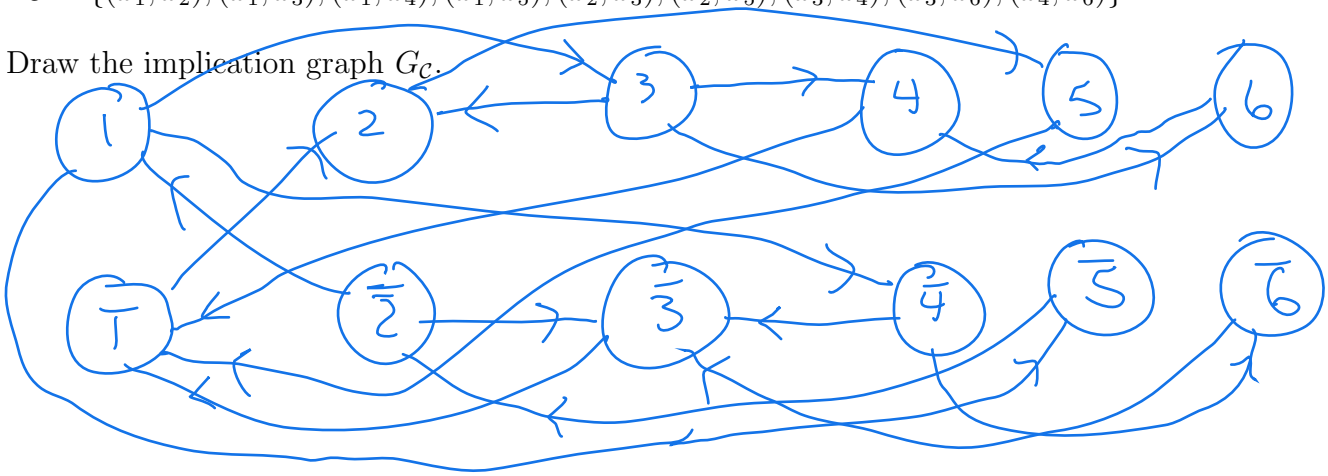
Wac(1,4) = min(0+wac(2,4), 60+Wac(3,4), Wac(1,2)+50, Wac(1,3)+0) + 180 =

min(210, 170, 190, 220) + 180 = 350 k=2

LO5. Consider the 2SAT instance

$$\mathcal{C} = \{(x_1, \overline{x}_2), (\overline{x}_1, x_3), (\overline{x}_1, \overline{x}_4), (\overline{x}_1, \overline{x}_5), (x_2, \overline{x}_3), (\overline{x}_2, x_5), (\overline{x}_3, x_4), (\overline{x}_3, x_6), (x_4, \overline{x}_6)\}.$$

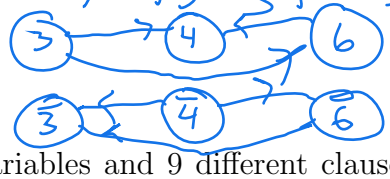
(a) Draw the implication graph $G_{\mathcal{C}}$.



(b) Perform the Improved 2SAT algorithm by computing the necessary reachability sets. Use numerical order (in terms of the variable index) and positive literal before negative literal when choosing the reachability set to compute next. Draw the resulting reduced 2SAT instance whenever a consistent reachability set is computed. Either provide a final satisfying assignment for $\mathcal{C}$ or indicate why $\mathcal{C}$ is unsatisfiable.

$R_{x_1} = \{x_1, x_3, x_2, x_6, \overline{x}_6, x_4, \overline{x}_1, \overline{x}_4, \overline{x}_3, \overline{x}_5, x_5, \overline{x}_2\}$

$R_{\overline{x}_1} = \{\overline{x}_1, x_2, x_5\}$ Reduced Graph:



$R_{x_3} = \{x_3, x_4, x_6\}$

$\alpha = (x_1 = 0, x_2 = \cdots = x_6 = 1)$

(c) A satisfiable instance $\mathcal{C}$ of 2SAT has 6 variables and 9 different clauses. When running the original 2SAT algorithm on this instance, a total of 8 queries were made before the algorithm returned 1. Based on this information, provide an example of an assignment α_1 that could possibly satisfy $\mathcal{C}$, as well as an assignment α_2 that definitely does *not* satisfy $\mathcal{C}$. Explain and justify your answers. Hint: you may assume that, for any variable x, the query reachable$(G_{\mathcal{C}}, x, \overline{x})$ performed first.

Possible Query Answers

From	To	Answer
x_1	$\overline{x}_1$	0
x_2	$\overline{x}_2$	0
x_3	$\overline{x}_3$	0
x_4	$\overline{x}_4$	0
x_5	$\overline{x}_5$	1
$\overline{x}_5$	x_5	0
x_6	$\overline{x}_6$	1
$\overline{x}_6$	x_6	0

$\Rightarrow$ Possible Assignment:

$\alpha_1 = (x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 1, x_5 = 0, x_6 = 0)$

Impossible Assignment:

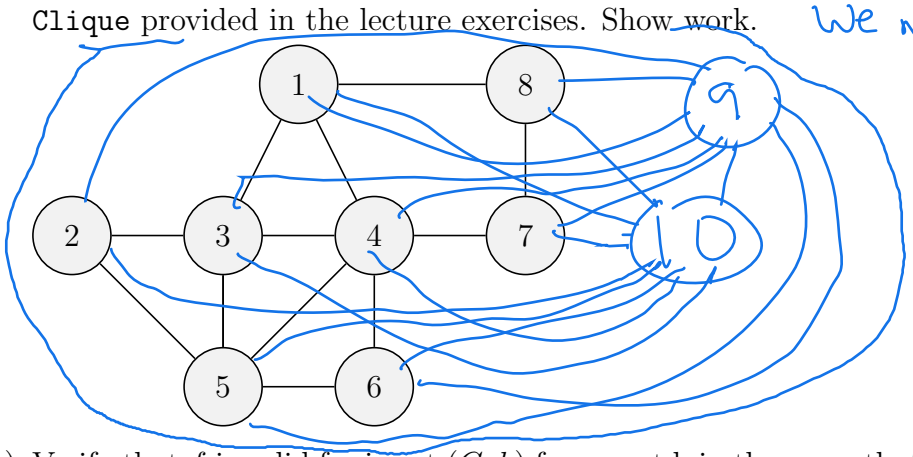
$(x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = 1)$

If all variables set to 1, it implies exactly 6 queries instead of 8.

LO6. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from problem A to problem B. (5 pts) See lecture Notes

(b) The simple graph $G = (V, E)$ shown below along with $k = 3$ is an instance of the `Clique` decision problem. Draw $f(G, k)$, where f is the mapping reduction from `Clique` to `Half Clique` provided in the lecture exercises. Show work.



f(G,K)= We must add n−2k = 8−6=2 new vertices and connect them to every other vertex.

(c) Verify that f is valid for input (G, k) from part b in the sense that both (G, k) and $f(G, k)$ are either both positive instances or both negative instances. Make sure your answer is specific to the instances from part b. (G,K) is positive for Clique since G has the 3-clique {1,3,4}. Similarly, f(G,K) has a half-clique {1,3,4,9,10} since

LO7. An instance of the `Max Cut` decision problem is a simple graph $G = (V, E)$ and an integer $k \geq 0$. The problem is to decide if there is a way to color the vertices of G using the colors red and blue and results in there being at least k edges $e = (u, v)$ for which u and v are assigned different colors.

|{1,3,4,9,10}| = 5 = 10/2.

(a) For a given instance (G, k) of `Max Cut`, describe a certificate in relation to (G, k).

$\alpha : V \to \{red, blue\}$ is a function that assigns each vertex the color red or blue.

(b) Provide a semi-formal verifier algorithm that takes as input i) an instance (G, k) of `Max Cut`, ii) a certificate for (G, k) as defined in part a, and decides if the certificate is valid for (G, k).

```
sum = 0
for each edge e = (u,v) ∈ E,
    If α(u) ≠ α(v), sum++.
Return (sum ≥ k).
```

(c) Use the size parameters $m = |E|$ and $n = |V|$ to describe the running time of your verifier from part b. Defend your answer in relation to the algorithm you provided for the verifier.

The verifier requires O(m) loop iterations, with each loop requiring O(1) steps (accessing values of array α). ∴ the verifier has O(m) running time.