CECS 528, Learning Outcome Assessment 9, 11/20/2024, Dr. Ebert

LO5. Consider the `2SAT` instance

$$\mathcal{C} = \{(x_1, \overline{x}_2), (\overline{x}_1, x_5), (\overline{x}_1, x_6), (x_2, \overline{x}_4), (\overline{x}_2, \overline{x}_3), (\overline{x}_2, \overline{x}_6), (x_3, \overline{x}_5), (x_4, \overline{x}_6), (\overline{x}_4, \overline{x}_6)\}.$$

(a) Draw the implication graph $G_{\mathcal{C}}$.

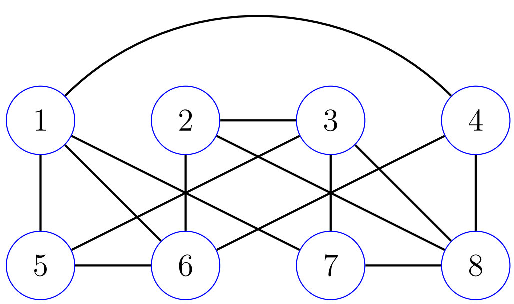
(b) Perform the `Improved 2SAT` algorithm by computing the necessary reachability sets. Use numerical order (in terms of the variable index) and positive literal before negative literal when choosing the reachability set to compute next. Draw the resulting reduced `2SAT` instance whenever a consistent reachability set is computed. Either provide a final satisfying assignment for $\mathcal{C}$ or indicate why $\mathcal{C}$ is unsatisfiable.

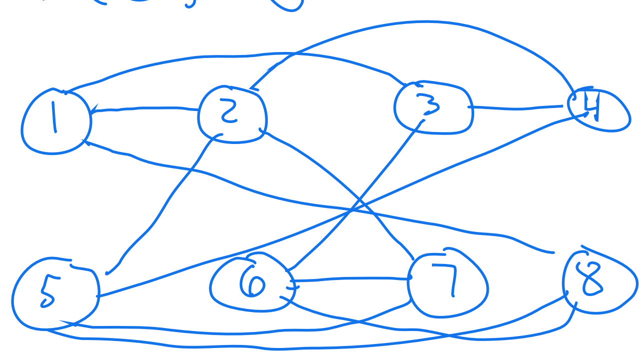
(c) Suppose `2SAT` instance $\mathcal{C}$ is satisfiable and the `Reachability`-oracle answers "yes" to `reachable`$(G_{\mathcal{C}}, x_3, \overline{x}_3)$. What can you say about the a satisfying assignment α for $\mathcal{C}$. Explain.

LO6. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from problem A to problem B.

(b) The simple graph $G = (V, E)$ shown below is an instance of the `Maximum Independent Set (MIS)` optimization problem. Draw $f(G)$, where f is the mapping reduction from `MIS` to `Maximum Clique` provided in lecture. $f(G) = \overline{G}$





(c) Verify that f is valid for input G of part b in the sense that both G and $f(G)$ have the same optimal solution. Hint: for both problems please assume that a "solution" is represented by a subset of vertices. Hint: G's max IS does *not* have a size equal to three.

G has a max IS equal to {2,4,5,7} while f(G) has a max clique equal to {2,4,5,7}✓.

LO7. An instance of the `Load Balancing (LB)` decision problem is a triple (a, p, k), where a is an array of n (not necessarily distinct) positive integers, and p and k are both positive integers. The problem is decide if there is a way to partition the array indices $0, 1, \ldots, n-1$ into p non-overlapping sets $P_1, \ldots, P_p$ so that, for each $0 \le i \le p$,

$$\sum_{j \in P_i} t[j] \le k.$$

The goal is to prove that `LB` is an `NP` decision problem.

(a) For given instance (a, p, k) of `LB` describe a certificate in relation to (a, p, k).

b is an array of size n whose members belong to $\{1, \ldots, p\}$, meaning b[i] indicates which processor to assign a[i].

(b) Provide a semi-formal verifier algorithm that takes as input i) an instance (a, p, k), and ii) a certificate for (a, p, k) as defined in part a, and decides if the certificate is valid for (a, p, k).

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For each i ∈ {1,...,p},
  sum = 0.
  For each j ∈ {1,...,n},
      If (b[j] == i) sum += a[j].
  If sum > k, return 0.
Return 1.
```

(c) Using size parameters n and b, where $b = \lfloor \log k \rfloor$ is the number of bits in the binary representation of k, prove that the number of steps taken by your verifier from part b is big-O of some polynomial with respect to n and b.

$O(npb)$

LO8. Recall the mapping reduction f from `3SAT` to `Subset Sum` described in lecture.

(a) An instance $\mathcal{C}$ of `3SAT` consists of clauses $c_1 = (x_1, x_2, \overline{x}_3)$, $c_2 = (x_1, \overline{x}_2, x_3)$, $c_3 = (\overline{x}_1, x_2, x_3)$, and $c_4 = (\overline{x}_1, x_2, \overline{x}_3)$. Draw the table of numbers associated with $f(\mathcal{C}) = (S, t)$.

See Lecture notes, but now upper-right quadrant has columns

y_1	1	1	0	0
	0	0	1	1
y_2	1	0	1	1
	0	1	0	0
	0	1	1	0
z_3	1	0	0	1

(b) For assignment $\alpha = (x_1 = 1, x_2 = 1, x_3 = 1)$ that satisfies $\mathcal{C}$, provide the associated subset $A \subseteq S$ that sums to target t, where $f(\mathcal{C}) = (S, t)$.

$A = \{y_1, y_2, y_3, \underline{g_1}, \underline{g_2}, \underline{g_3}, \underline{g_4}, \underline{h_4}\}$

$\sum_{a \in A} a = t = 1{,}111\ 3{,}333$

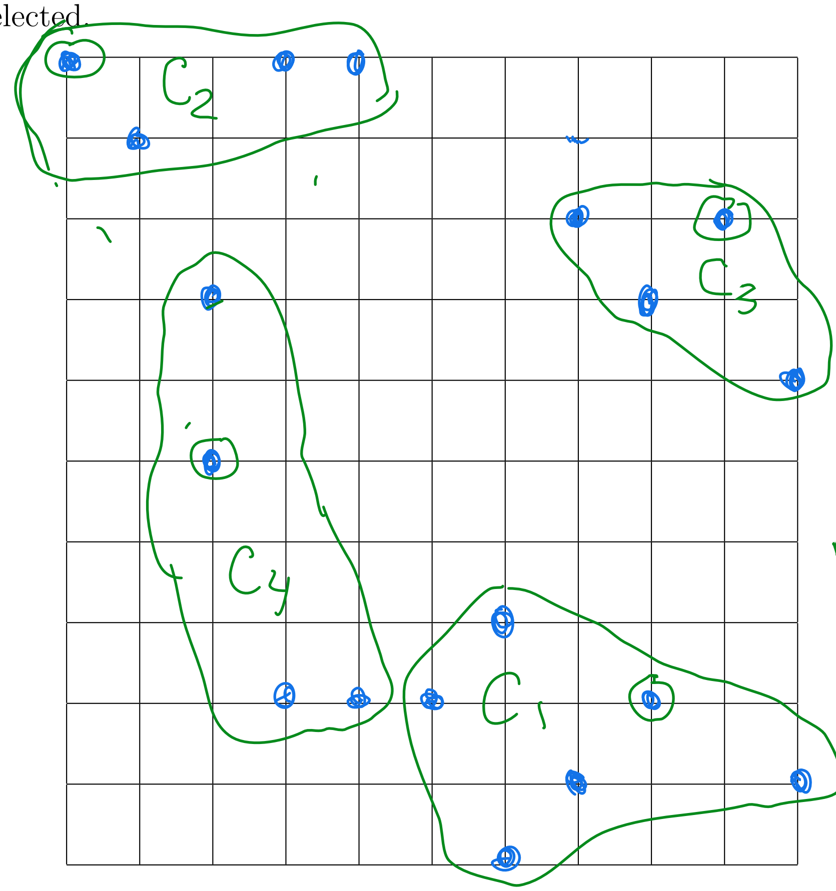
LO9. Do the following.

(a) Draw the set

$$S = \{(0,10),(1,9),(2,5),(2,7),(3,2),(3,10),(4,2),(4,10),(5,2),(6,0),$$
$$(6,3),(7,1),(7,8),(8,2),(8,7),(9,8),(10,1),(10,5)\}$$

of points and apply the **k-Clustering** algorithm to S and $k = 4$. Select $(8,2)$ as the first center. Clearly indicate which points are cluster centers enclose each cluster with a boundary curve. Label each cluster (e.g. C_1) in accordance with the order in which it's center was selected.

Centers = $\{(8,2), (0,10), (9,8)\,(2,5)\}$



max diameter: $\sqrt{29}$ for C_1

(b) Explain the reasoning behind why the **Vertex Cover** approximation algorithm has an approximation ratio of at most 2.

When encountering an edge (u, v) for which neither u nor v is in the current cover we know that at least one should be in the cover, and so adding both means, in the worst case, we've added at most twice the amount that was actually needed.