

CECS 528, Exam 1 Part a, Fall 2024, Dr. Ebert

Directions: show all work.

Problem

LO4. Answer the following.

- (a) The dynamic-programming algorithm that solves the 0-1 Knapsack optimization problem defines a recurrence for the function $p(i, c)$. In words, what does $p(i, c)$ equal? Hint: do not write the recurrence (see Part b). (5 pts)

See Lecture Notes

- (b) Provide the dynamic-programming recurrence for $p(i, c)$. (8 pts)

See Lecture Notes

- (c) Apply the recurrence from Part b to a knapsack having capacity $M = 10$ and items

item	weight	profit
1	5	30
2	4	30
3	1	20
4	4	40
5	5	30
6	5	60

$$20 + 40 + 60 = 120$$

Show the matrix of subproblem solutions and use it to provide an optimal set of items.

(12 pts)

PC(i, c)

i \ c	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	30	30	30	30	30	30
2	0	0	0	0	30	30	30	30	30	60	60
3	0	20	20	20	30	50	50	50	50	60	80
4	0	20	20	20	40	60	60	60	70	90	90
5	0	20	20	20	40	60	60	60	70	90	90
6	0	20	20	20	40	60	80	80	80	90	120

Items Added: 6, 4, 3

A1. Given recurrence $T(n) = aT(n/b) + f(n)$, for Case 3 of the Master Theorem to apply, one requirement that we have mostly ignored (because it's always true when f is comprised of a power function times a log power function) is that there must exist a constant $c < 1$ for which

$$af(n/b) \leq cf(n).$$

Determine a value for $c < 1$ that satisfies the above inequality in the case that $f(n) = n^3 \log n$, $a = 4$, and $b = 2$. Show all work and justify your answer. (35 pts)

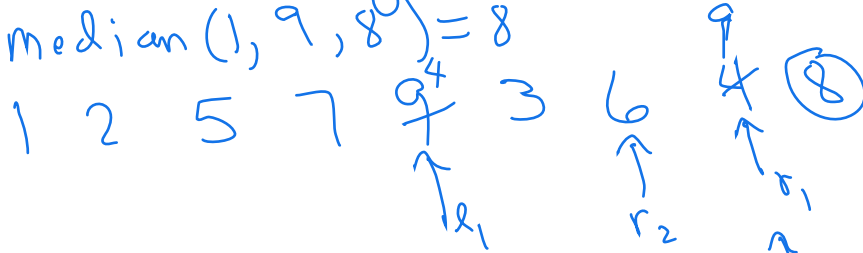
$$\begin{aligned}
 4 \left(\frac{n}{2}\right)^3 \log\left(\frac{n}{2}\right) &\leq c n^3 \log n \Leftrightarrow \\
 \frac{n^3}{2} (\log n - 1) &\leq c n^3 \log n \Leftrightarrow \\
 \frac{1}{2} \left(1 - \frac{1}{\log n}\right) &\leq c.
 \end{aligned}$$

$\therefore C = \frac{1}{2}$ satisfies the condition.

A2. Provide a permutation of the numbers 1-9 so that, when sorted by Quicksort using median-of-three heuristic, the a_{right} subarray always has one element in rounds 1, 2, and 3. Verify that your array is correct by demonstrating Quicksort for each of the three rounds. (35 pts)

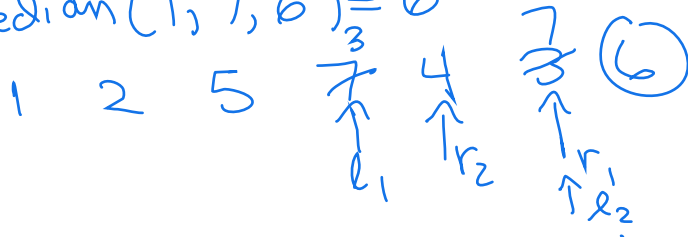
Consider the array $a = 1\ 2\ 5\ 7\ 9\ 3\ 6\ 4\ 8$

Round 1: $\text{median}(1, 9, 8) = 8$



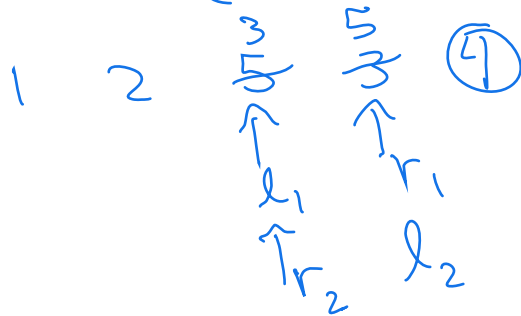
$a_{\text{left}} = 1\ 2\ 5\ 7\ 4\ 3\ 6$ $a_{\text{right}} = 9$

Round 2: $\text{median}(1, 7, 6) = 6$



$a_{\text{left}} = 1\ 2\ 5\ 3\ 4$ $a_{\text{right}} = 7$

Round 3: $\text{median}(1, 5, 4) = 4$



$a_{\text{left}} = 1\ 2\ 3$ $a_{\text{right}} = 5$