CECS 528, Exam 1 Part a, Fall 2024, Dr. Ebert

Directions: show all work.

Problem

LO4. Answer the following.

(a) The dynamic-programming algorithm that solves the `0-1 Knapsack` optimization problem defines a recurrence for the function $p(i, c)$. In words, what does $p(i, c)$ equal? Hint: do *not* write the recurrence (see Part b). (5 pts)

See Lecture Notes

(b) Provide the dynamic-programming recurrence for $p(i, c)$. (8 pts)

See Lecture Notes

(c) Apply the recurrence from Part b to a knapsack having capacity $M = 10$ and items

item	weight	profit
1	5	30
2	4	30
3	1	20
4	4	40
5	5	30
6	5	60

$20 + 40 + 60 = 120$

Show the matrix of subproblem solutions and use it to provide an optimal set of items. (12 pts)

$P(i, c)$

i \ c	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	30	30	30	30	30	30
2	0	0	0	0	30	30	30	30	30	60	60
3	0	20	20	20	30	50	50	50	50	60	80
4	0	20	20	20	40	60	60	60	70	90	90
5	0	20	20	20	40	60	60	60	70	90	90
6	0	20	20	20	40	60	80	80	80	90	120

Items Added: 6, 4, 3

A1. Given recurrence $T(n) = aT(n/b) + f(n)$, for Case 3 of the Master Theorem to apply, one requirement that we have mostly ignored (because it's always true when f is comprised of a power function times a log power function) is that there must exist a constant $c < 1$ for which

$$af(n/b) \leq cf(n).$$

Determine a value for $c < 1$ that satisfies the above inequality in the case that $f(n) = n^3 \log n$, $a = 4$, and $b = 2$. Show all work and justify your answer. (35 pts)

$$4\left(\frac{n}{2}\right)^3 \log\left(\frac{n}{2}\right) \leq cn^3 \log n \iff$$

$$\frac{n^3}{2}(\log n - 1) \leq cn^3 \log n \iff$$

$$\frac{1}{2}\left(1 - \frac{1}{\log n}\right) \leq c.$$

$\therefore c = \frac{1}{2}$ satisfies the condition.

A2. Provide a permutation of the numbers 1-9 so that, when sorted by Quicksort using median-of-three heuristic, the a_{right} subarray always has one element in rounds 1,2, and 3. Verify that your array is correct by demonstrating Quicksort for each of the three rounds. (35 pts)

Consider the array $a = 1\ 2\ 5\ 7\ 9\ 3\ 6\ 4\ 8$

Round 1: median(1, 9, 8) = 8

$1\ 2\ 5\ 7\ \cancel{9}^{4}\ 3\ 6\ \cancel{4}^{9}\ 8$

$\ell_1 \quad r_2 \quad r_1 \quad \ell_1$

$a_{\text{left}} = 1\ 2\ 5\ 7\ 4\ 3\ 6 \qquad a_{\text{right}} = 9$

Round 2: median(1, 7, 6) = 6

$1\ 2\ 5\ \cancel{7}^{3}\ 4\ \cancel{3}^{7}\ 6$

$\ell_1 \quad r_2 \quad r_1 \quad \ell_2$

$a_{\text{left}} = 1\ 2\ 5\ 3\ 4 \qquad a_{\text{right}} = 7$

Round 3: median(1, 5, 4) = 4

$1\ 2\ \cancel{5}^{3}\ \cancel{3}^{5}\ 4$

$\ell_1 \quad r_1 \quad r_2 \quad \ell_2$

$a_{\text{left}} = 1\ 2\ 3 \qquad a_{\text{right}} = 5$