CECS 528, Exam 1 Part a, Fall 2024, Dr. Ebert

Directions: show all work.

Problem

LO4. Answer the following.

(a) The dynamic-programming algorithm that solves the `0-1 Knapsack` optimization problem defines a recurrence for the function $p(i, c)$. In words, what does $p(i, c)$ equal? Hint: do *not* write the recurrence (see Part b). (5 pts)

(b) Provide the dynamic-programming recurrence for $p(i, c)$. (8 pts)

(c) Apply the recurrence from Part b to a knapsack having capacity $M = 10$ and items

item	**weight**	**profit**
1	5	30
2	4	30
3	1	20
4	4	40
5	5	30
6	5	60

Show the matrix of subproblem solutions and use it to provide an optimal set of items. (12 pts)

A1. Given recurrence $T(n) = aT(n/b) + f(n)$, for Case 3 of the Master Theorem to apply, one requirement that we have mostly ignored (because it's always true when f is comprised of a power function times a log power function) is that there must exist a constant $c < 1$ for which

$$af(n/b) \leq cf(n).$$

Determine a value for $c < 1$ that satisfies the above inequality in the case that $f(n) = n^3 \log n$, $a = 4$, and $b = 2$. Show all work and justify your answer. (35 pts)

A2. Provide a permutation of the numbers 1-9 so that, when sorted by Quicksort using median-of-three heuristic, the a_{right} subarray always has one element in rounds 1,2, and 3. Verify that your array is correct by demonstrating Quicksort for each of the three rounds. (35 pts)