CECS 329, Homework Assignment 6, Fall 2025, Dr. Ebert

Directions: Please review the `Homework` section on page 6 of the syllabus including a list of all rules and guidelines for writing and submitting solutions, This includes the statement regarding plagiarism.

Due Date: Saturday, December 13th as a PDF-file upload to the HW6 Canvas dropbox.

Problems

Note: a total point score of at least 50 is necessary for an LO11 pass (not including Problem 3a).

1. Convert the CFG G whose rules are listed below to an equivalent CFG whose rules are in Chomsky normal form.

$$S \to Tc$$
$$T \to aTc \mid Tc \mid U$$
$$U \to Ubb \mid \varepsilon.$$

   Note that S is the start symbol and that all variables are capitalized, while terminals are in lowercase. For each step, peform exactly one transformation (e.g. new start symbol,a single ε-rule removal, or a single unit-rule removal). The only exception is that you may define all (capitalized please!) new variables in a single step that includes transforming existing rules with those new variables (this should be your final step). Remember that the removal of an ε-rule may create another ε-rule that must be subsequently removed. The same is true for unit rules of the form $X \to Y$, where X and Y are variables. (20 pts)

2. Recall the recursive `accept` procedure described in the proof that $\texttt{Accept}_{\texttt{CFG}}$ is decidable. Draw the recursion tree that arises during the computation of `accept` on inputs G, $w = abbb$, and $\nu = S$, where CFG G has the rules

$$S \to TB$$
$$T \to AB \mid UB \mid WB \mid b$$
$$U \to UB \mid b.$$
$$W \to AT$$
$$A \to a$$

   and

$$B \to b.$$

   Label each node of the tree with both the terminal-word and variable-word inputs that occur at that point of the recursion. For example, the root node should be labeled as $(S, abbb)$. When substituting for the first variable of ν, follow the order of the bodies listed for that variable. For example, if the variable is T, then the order of substitution would be AB, UB, WB, and b. (15 pts)

3. Do the following.

   (a) If w is a word over some alphabet, then w^r denotes the **reversal** of w and its letters are those of w, but written in reverse order. For example,

   $$(\text{happy})^r = \text{yppah}.$$

   Extending this operation to a language, the **reversal of language L** is the language consisting of the set of words

   $$\{w^r \mid w \in L\}.$$

   In other words, its the set of all words of L but in reverse order. Use structural induction to prove that, if L is regular, then so is L^r. See pages 2 and 3 of the lecture "On the Equivalence Between Regular Languages and Regular Expressions". In other words, the idea is to use structural induction with respect to the structure of the regular expression that describes L. (20 pts) Note: the points earned for this problem counts for HW6 points, but does *not* count for LO11 points.

   (b) A language L is said to be **closed under reversal** iff, for every $w \in L$, w^r is also in L. For example, the language $\{001, 011, 100, 110, 1, 11\}$ is closed under reversal, but not the language $\{01, 100, 111\}$ since 01 is in the language, but not $(01)^r = 10$. Now suppose L is some regular language. Prove that the problem of determining whether or not L is closed under reversal is in fact decidable. (20 pts)

4. An instance of the `Happy Birthday` decision problem is a Turing machine M (whose input alphabet Σ includes, but is *not* limited to all lowercase letters) and a word w over Σ. The problem is to decide if, at any time during the computation of M on input w, the word "happy birthday" is spelled somewhere on the tape (with a blank cell separating the y from the b). Prove that `Happy Birthday` is an undecidable problem. (20 pts)