

CECS 329, Exam 2 Spring 2025, Dr. Ebert

IMPORTANT: READ THE FOLLOWING DIRECTIONS. Directions,

- For each of LO's 4-8 and the Additional problem, write your solution using a **SINGLE SHEET OF PAPER ONLY (BOTH FRONT AND BACK)**. Write **NAME** and **PROBLEM NUMBER** on each sheet.
- For Unit-1 LO's it's OK to use the same sheet for two or more problems if there is sufficient space.

Unit-2 LO's (25 Points Each)

LO4. Do the following.

- a. Consider the 3SAT instance

$$\mathcal{C} = \{c_1 = (x_1, x_2, x_3), c_2 = (\bar{x}_1, \bar{x}_2, \bar{x}_3), c_3 = (\bar{x}_1, x_2, x_3), c_4 = (x_1, \bar{x}_2, \bar{x}_3)\}.$$

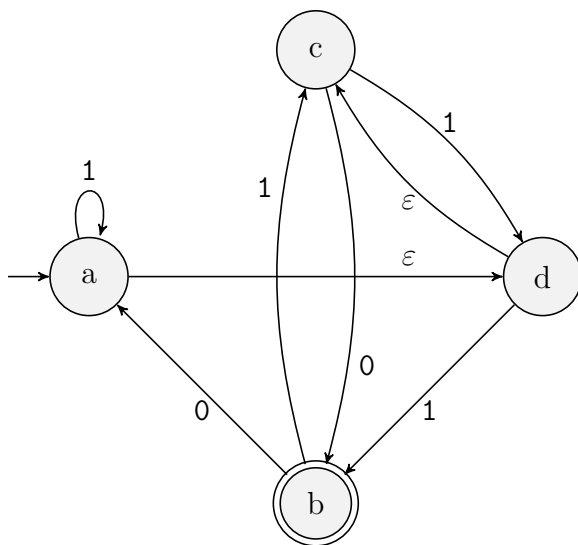
Consider the mapping reduction $f : \text{3SAT} \rightarrow \text{Subset Sum}$ from 3SAT to Subset Sum that was presented in lecture.

- Draw the complete table associated with $f(\mathcal{C}) = (S, t)$. (8 pts)
 - Verify that $\mathcal{C}$ is satisfiable and use the satisfying assignment α to construct a corresponding subset A of S that sums to t . Please list the members of A using their alphabetical names (rather than their numerical values). (10 pts)
- b. Draw the parse tree for $F(x_1, x_2, x_3) = (x_1 \vee \bar{x}_2) \vee (\bar{x}_2 \wedge x_3)$, label the internal nodes with appropriate variables, and use them to provide the initial formula that begins the Tseytin transformation from F to an instance of 3SAT. Do *not* simplify the formula. (7 pts)

LO5. Do the following.

- Let L be the language of binary words that contain an odd number of 1's and exactly two 0's. Provide the state diagram for a DFA M that accepts L . (12 pts)
- For each state s that appears in your DFA from part a, provide a sentence that describes what must be true about *any* word that, when read by M causes M to finish its computation in state s . (7 pts)
- Demonstrate the computation of M on inputs i) $w_1 = 01101$ and ii) $w_2 = 10001$. For each computation, indicate whether w is accepted or rejected. (6 pts)

LO6. For the NFA N whose state diagram is shown below, provide a table that represents N 's δ transition function.

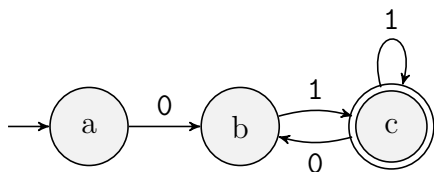


- Provide a table that represents N 's δ transition function. (12 pts)
- Use the table from part a to convert N to an equivalent DFA M using the method of subset states. Draw M 's state diagram. (8 pts)
- Show the computation of M on input $w = 11001$. (5 pts)

LO7. Do the following.

- Let L_1 denote the language of binary words that contain exactly two 0's and an odd number of 1's and, L_2 denote the language consisting of all binary words w that have at least two 0's and at most two 1's. Use set notation to write the members of $L_1 \cap L_2$. (6 pts)
- Is 1000101010010 a member of L_1^* ? Explain. (6 pts)
- Provide a regular expression that represents the language consisting of all binary words w for which the first bit does not equal the last bit, and there are exactly two 0's. (13 pts)

LO8. Consider the NFA N shown below.



and let $L(N)$ denote the language that it recognizes.

- Use N to construct the NFA N' that accepts $(L(N))^*$ and uses the algorithm that converts an NFA that accepts L , to one that accepts L^* . (10 pts)
- Demonstrate each step of the GNFA-to-Regular-Expression algorithm that computes a regular expression that describes $L(N)$. Hint: your initial GNFA should have four states. (15 pts)

Additional Problem

Answer the following.

1. What does it mean for a decision problem to be NP-Complete? Provide the official definition. (10 pts)
2. Provide the chain of mapping reductions that establishes the NP-completeness of the **Traveling Salesperson** decision problem. Hint: $\text{SAT} \leq_m^p \text{3SAT}$ is the first link of the chain. (8 pts)
3. Professor Jones has just discovered a polynomial-step algorithm for deciding any instance of the **Independent Set** decision problem. Explain why this proves that $P = NP$ (12 pts)

Unit-1 LO's (0 Points Each)

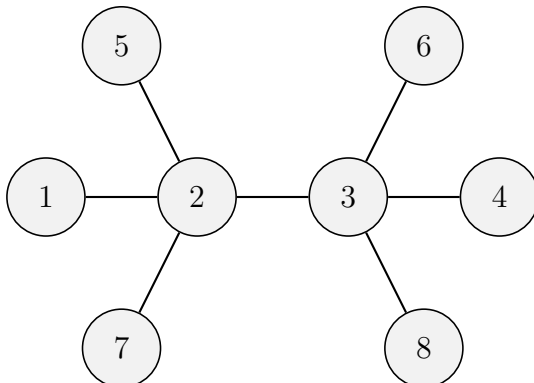
LO1. Consider the 2SAT instance

$$\mathcal{C} = \{(\bar{x}_1, x_4), (x_1, \bar{x}_5), (\bar{x}_2, \bar{x}_3), (\bar{x}_2, x_4), (x_2, x_6), (x_3, x_4), (\bar{x}_3, x_6), (\bar{x}_4, \bar{x}_5), (\bar{x}_4, x_5)\}.$$

- (a) Draw the implication graph $G_{\mathcal{C}}$.
- (b) Perform the **Improved 2SAT** algorithm by computing the necessary reachability sets. Use numerical order (in terms of the variable index) and positive literal before negative literal when choosing the reachability set to compute next. Draw the resulting reduced 2SAT instance whenever a consistent reachability set is computed. Either provide a final satisfying assignment for $\mathcal{C}$ or indicate why $\mathcal{C}$ is unsatisfiable.
- (c) If an instance $\mathcal{C}$ has 336 variables and 2027 clauses, then what is the least number of queries that must be made to the **Reachability** oracle in order to confirm that $\mathcal{C}$ is satisfiable? Explain.

LO2. Answer the following.

- (a) Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B .
- (b) For the mapping reduction $f : \text{Vertex Cover} \rightarrow \text{Half Vertex Cover}$, draw $f(G, k)$ for the **Vertex Cover** instance whose graph is shown below, and for which $k = 2$.



- (c) Verify that both (G, k) and $f(G, k)$ are either both positive instances, or are both negative ones. Explain and show work.

LO3. An instance of **Feedback Arc Set (FAS)** is a directed graph $G = (V, E)$ and a natural number $k \geq 0$. The problem is to decide if there is a set S of k vertices of G for which, when removing the vertices of S from G (and all edges incident with them) the resulting graph is acyclic. To see that **FAS** is an NP problem we define a certificate for instance (G, k) to be a set $S \subseteq V$ of k vertices. The following pseudocode is used by the verifier to determine if S is in fact a set of vertices whose removal from G creates an acyclic graph. Note: a minimum of 18 points is needed to pass this LO.

$G' = (V', E')$, where $V' = V - S$ and $E' = \{(u, v) \in E \mid u, v \in V'\}$.

For each $u \in V'$,

 For each $v \in V'$,

 If $(u, v) \in E'$ and $\text{Reachable}(G', v, u)$, then return 0.

 // v can reach u and so there is a cycle that includes v

Return 1.

- (a) Provide size parameters for the **FAS** problem and describe what each represents in relation to an **FAS** problem instance. Hint: there are two of them. (6 pts)
- (b) Use the size parameters to provide the big-O number of steps that is required by the verifier to check the validity of a certificate. Hint: a single call to function Reachable requires a linear number of steps with respect to the size parameters of G . Justify your answer. (7 pts)
- (c) Classify each of the following problems as being in P, NP, or co-NP (3 points each).
 - i. An instance of **Fallible** is a Boolean formula F . The problem is to decide if there is an assignment that can be made to the variables of F so that F evaluates to 0.
 - ii. An instance of **Mine Sweep** is a simple graph $G = (V, E, f)$ where $f : V \rightarrow \{-1, 0, 1, \dots\}$ is a function from the set of vertices to the set of natural numbers, including -1. If $f(v) \geq 0$, then it means that a total of $f(v)$ neighbors of v (i.e., vertices that are adjacent to v) must have a mine placed on them. On the other hand, if $f(v) = -1$, then there is no constraint on how many neighbors of v must have a mine. The problem is to decide if there is a function $g : V \rightarrow \{0, 1\}$, such that i) $g(v)$ indicates whether or not a mine is placed on vertex v , and ii) for all $v \in V$, if $f(v) \geq 0$, then

$$f(v) = \sum_{u \in N(v)} g(u),$$

where $N(v)$ is the set of all neighbors of v . In other words, function g meets all the mine constraints that are indicated by f .

- iii. An instance of **Palindrome** is an array a of integer, and the problem is to decide if a reads the same forwards as backwards, meaning that, for all $i \in \{0, 1, \dots, n-1\}$, $a[i] = a[n-1-i]$.
- iv. An instance of **Large Vertex Covers** is a graph $G = (V, E)$, and the problem is to decide if every vertex cover of G has size at least $n/2$, where $n = |V|$.