CECS 329, Quiz 1, Friday, Fall 2025, Dr. Ebert

IMPORTANT: READ THE FOLLOWING DIRECTIONS.

- For each problem, write your solution using **ONE SHEET OF PAPER ONLY (BOTH FRONT AND BACK)**. Write **NAME** and **PROBLEM NUMBER** on each sheet.
- Write solutions to different problems on **SEPARATE SHEETS** of paper.

Unit 2 LO Problems

LO4. Answer the following.

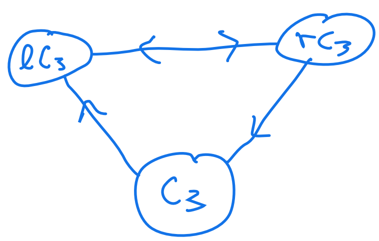
(a) Consider the `3SAT` instance

$$\mathcal{C} = \{c_1 = (x_1, x_2, x_3), c_2 = (\overline{x}_1, \overline{x}_2, \overline{x}_3), c_3 = (\overline{x}_1, x_2, x_3), c_4 = (x_1, \overline{x}_2, \overline{x}_3)\}.$$

and the mapping reduction $f : \texttt{3SAT} \to \texttt{DHP}$ from `3SAT` to `Directed Hamilton Path` that was presented in lecture.

i. Consider the vertices labeled as lc_3 and rc_3 that appear in the x_2-diamond chain of graph $G = f(\mathcal{C})$. Draw each of these vertices along with vertex c_3 and draw the edges that exist between these three vertices. Explain why your drawing is correct. Hint: one of the edges is bidirectional.

Solution. The drawing below is correct because clause c_3 is satisfied when assigning x_2 the value 1. Hence, we want to be able to visit c_3 from the x_2-diamond only if we are moving from right to left.



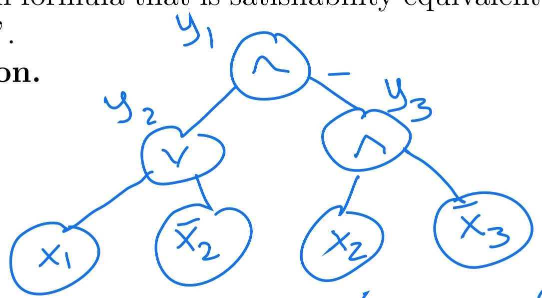
ii. Does G have a Hamilton Path? Defend your answer. If yes, then provide a **trip itinerary** for such a path, meaning a description of the direction of travel (left-to-right or right-to-left) through each diamond, and which clauses are visited in each diamond.

Solution. Yes, since $\alpha = (x_1 = 1, x_2 = 0, x_3 = 1)$ satisfies $\mathcal{C}$, $f(\mathcal{C}) = (G, a, b)$ is such that G has an HP from a to b. Moreover, a trip itinerary that produces the HP involves going right-to-left in diamond x_1, left-to-right in diamond x_2, and right-to-left in diamond x_3. Finally, c_1 and c_4 are visited from diamond x_1, c_2 from diamond x_2, and c_3 from diamond x_3.

(b) Answer the following regarding the mapping reduction from `SAT` to `3SAT` that uses the Tseytin transformation.

i. If the Boolean formula $F(x_1, x_2, x_3) = (x_1 \vee \overline{x}_2) \wedge (x_2 \wedge \overline{x}_3)$, then provide the initial Boolean formula that is satisfiability equivalent to F. Hint: this formula uses "double arrows".

Solution.



$$y_1 \wedge (y_1 \leftrightarrow (y_2 \wedge y_3)) \wedge (y_2 \leftrightarrow (x_1 \vee \overline{x}_2)) \wedge (y_3 \leftrightarrow (x_2 \wedge \overline{x}_3))$$

ii. By using the steps of the Tseytin transformation described in lecture, show how to convert the Boolean formula $y \leftrightarrow (x_1 \wedge \overline{x}_3)$ to a collection of `3SAT` clauses. Hint: this part is unrelated to the formula F described in part i.

$$y \leftrightarrow (x_1 \wedge \overline{x}_3) \iff (y \rightarrow (x_1 \wedge \overline{x}_3) \wedge ((x_1 \wedge \overline{x}_3) \rightarrow y)) \iff$$

$$(\overline{y} \vee (x_1 \wedge \overline{x}_3)) \wedge (\overline{(x_1 \wedge \overline{x}_3)} \vee y) \iff$$

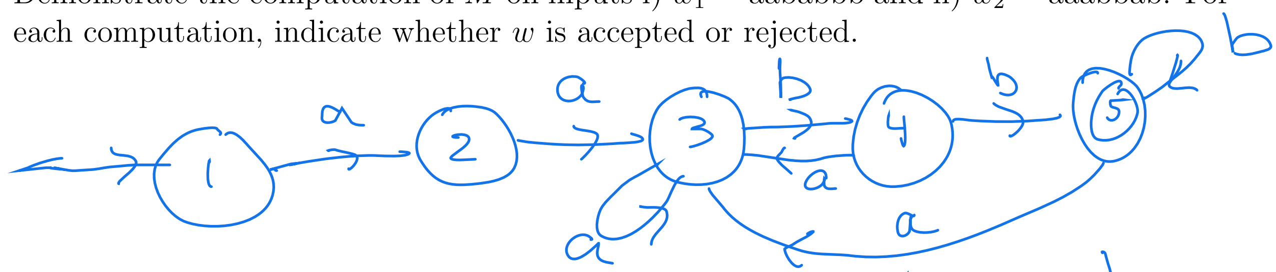
$$(\overline{y} \vee x_1) \wedge (\overline{y} \vee \overline{x}_3) \wedge (\overline{x}_1 \vee x_3 \vee y) \iff$$

$$\{(\overline{y}, x_1, x_1), (\overline{y}, \overline{x}_3, \overline{x}_3), (\overline{x}_1, x_3, y)\}$$

LO5. Do the following.

(a) Consider the language L of all words w over the alphabet {a,b} for which w begins with aa and ends with bb. Provide a DFA M that accepts L.

(b) Demonstrate the computation of M on inputs i) $w_1 =$ aababbb and ii) $w_2 =$ aaabbab. For each computation, indicate whether w is accepted or rejected.



a	a	b	a	b	b	b	
1	2	3	4	3	4	5	$\boxed{5}$

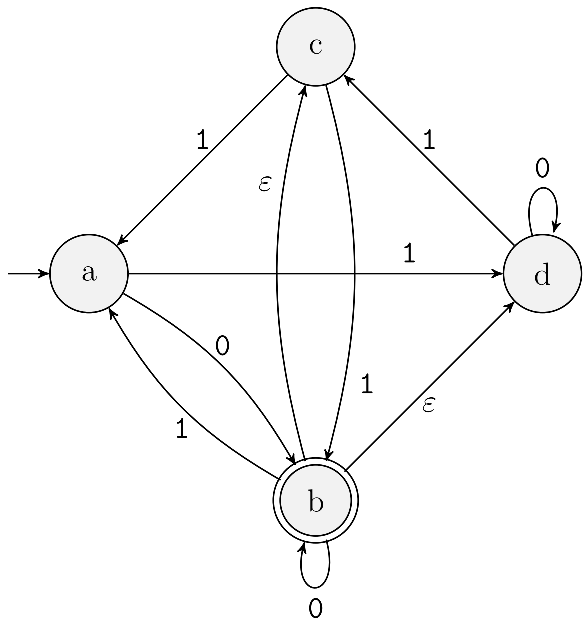
$\Downarrow$ accept

a	a	a	b	b	a	b	
1	2	3	3	4	5	3	$\boxed{4}$

$\Downarrow$ reject

LO6. Do the following.

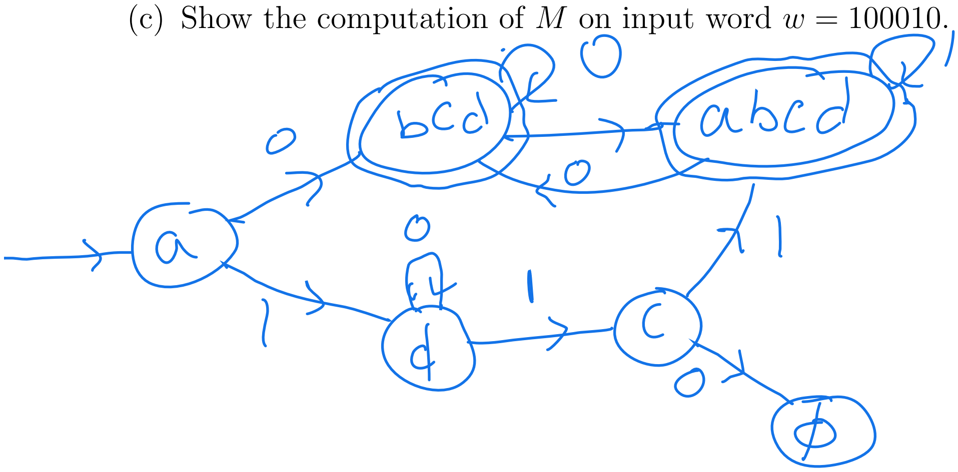
(a) For the NFA N whose state diagram is shown below, provide a state transition table for N.



	0	1
a	bcd	d
b	bcd	a
c	$\emptyset$	abcd
d	d	c

(b) Use the table from part b to convert N to an equivalent DFA M using the method of subset states. Draw M's state diagram.

(c) Show the computation of M on input word $w = 100010$. Is w accepted?



$a \xrightarrow{1} d \xrightarrow{0} d \xrightarrow{0} d \xrightarrow{0} d \xrightarrow{1} c \xrightarrow{0} \boxed{\emptyset}$

reject

Makeup Problems

LO1. Do/answer the following.

(a) Draw the implication graph $G_{\mathcal{C}}$ associated with the 2SAT instance

$$\mathcal{C} = \{(\overline{x}_1, x_4), (x_1, \overline{x}_5), (\overline{x}_2, \overline{x}_3), (\overline{x}_2, x_4), (x_2, x_6), (x_3, x_4), (\overline{x}_3, x_6), (\overline{x}_4, \overline{x}_5), (\overline{x}_4, x_5)\}.$$

(b) Perform the `Improved 2SAT` algorithm by computing the necessary reachability sets. Use numerical order (in terms of the variable index) and positive literal before negative literal when choosing the reachability set to compute next. Draw the resulting reduced `2SAT` instance whenever a consistent reachability set is computed. Either provide a final satisfying assignment for $\mathcal{C}$ or indicate why $\mathcal{C}$ is unsatisfiable.

Solution. $\mathcal{C}$ is satisfiable, and a satisfying assignment α produced by consistent reachability set

$$R_{\overline{x}_1} = \{\overline{x}_1, \overline{x}_2, x_3, \overline{x}_4, \overline{x}_5, x_6\}$$

so that $\alpha = (x_1 = x_2 = 0, x_3 = 1, x_4 = x_5 = 0, x_6 = 1)$.

(c) If an instance $\mathcal{C}$ has 336 variables and 2027 clauses, then what is the worst-case number of queries that must be made to the `Reachability` oracle in order to confirm that $\mathcal{C}$ is unsatisfiable? Explain.

Solution. In the worst case $336 \times 2 = 772$ queries are needed. For x_i, $i = 1, \ldots, 335$, we would have reachable$(G_{\mathcal{C}}, x_i, \overline{x}_i) = 1$, while reachable$(G_{\mathcal{C}}, \overline{x}_i, x_i) = 0$, followed by reachable$(G_{\mathcal{C}}, x_{336}, \overline{x}_{336}) = 1$ and reachable$(G_{\mathcal{C}}, \overline{x}_{336}, x_{336}) = 1$, for a total of 772 queries.

LO2. Do the following.

(a) Provide the definition of what it means to be a mapping reduction from problem A to problem B. Hint: do *not* assume that A and B are decision problems.

(b) Recall the mapping reduction $f : \texttt{SP} \to \texttt{SS}$ from `Set Partition` to `Subset Sum`. Compute $f(S)$ in case $S = \{5, 11, 13, 23\}$.

Solution. $f(S) = (S, t = 26)$ where 26 is one half the sum of the members of S.

(c) Verify that f is a valid mapping reduction for input S of part b in the sense that both S and $f(S)$ are either both positive instances or both negative instances of their respective problems. Your answer should be specific to S provided above. For example, if both are positive (respectively, negative) instances, then provide evidence of this.

Solution. By inspection, we see that no subset of S can sum to 26. Hence, there is no way to partition S into two sets that each sum to 26, and both S and $f(S)$ are negative instances of their respective problems.

LO3. An instance $\mathcal{C}$ of `CNF-SAT` is a set of clauses $\{c_1, \ldots, c_m\}$ where each clause c_i, $i = 1, \ldots, m$, is of the form $l_{i1} \vee \cdots \vee l_{ik_i}$, where $1 \le k_i \le n$ and each l_{ij}, $1 \le j \le k_i$, is a literal from the set $\{x_1, \overline{x}_1, \ldots, x_n, \overline{x}_n\}$, $1 \le k_i \le n$. The problem is to decide if there is an assignment α over variables $\{x_1, \ldots, x_n\}$ for which α satisfies each clause c_i, meaning that $\alpha(l_{ij}) = 1$ for some $1 \le j \le k_i$. Thus, a certificate for `CNF-SAT` is such an assignment α. The following pseudocode is used by a verifier that decides if assignment α satisfies instance $\mathcal{C}$.

For each $i \in \{1, \ldots, m\}$

  found = 0. //We have yet to find a literal of c_i that is assigned 1.

  For each $j \in \{1, \ldots, k_i\}$

    If $\alpha(l_i) = 1$,

    · found = 1 //Found a literal l_{ij} of c_i that is assigned 1.

    · Break out of the inner loop.

  If found = 0, return 0.

Return 1.

(a) Provide size parameters for the `CNF-SAT` problem and describe what each represents in relation to a `CNF-SAT` problem instance. Hint: there are two of them. (6 pts)

Solution. $m = |\mathcal{C}|$ is the number of clauses of $\mathcal{C}$ and $n = |V|$ is the number of variables that appear in the clauses of $\mathcal{C}$,

(b) Use the size parameters to provide the big-O number of steps that is required by the verifier to check the validity of a certificate. Justify your answer. (7 pts)

Solution. The outer loop iterates at most m times and, for each such iteration, the inner loop iterates at most n times, since a clause can have at most n literals. Moreover, assuming that α is an array, the number of steps required to execute the inner loop is O(1). Therefore, the number of steps equals $O(mn)$ and the verifier requires a quadratic number of steps.

(c) Classify each of the following problems as being in `P`, `NP`, or co-`NP` (3 points each).

Solution. P, co-NP, co-NP, NP

i. An instance of `One Subsequence Sum (OSS)` is an array of integers a and the problem is to decide if there are indices $0 \le i < j < n$ for which

$$a[i] + a[i+1] + \cdots + a[j] = 1.$$

ii. An instance of `Subset Sum Avoidance (SSA)` is a pair (S, t), where S is a set of natural numbers and $t \ge 0$ is a natural number. The problem is to decide if all subsets $A \subseteq S$ have the property that

$$\sum_{a \in A} a \ne t.$$

iii. An instance of `Fallacy` is a Boolean formula $F(x_1, \ldots, x_n)$ and the problem is to decide if F is a fallacy, meaning that $F(\alpha) = 0$ for every assignment α over the variables $x_1, \ldots, x_n$.

iv. An instance of `Word Cover` is a pair (S, k), where $S = \{w_1, \ldots, w_n\}$ is a set of n binary words, each having fixed length $m \geq 1$, and $k \geq 0$ is a natural number. The problem is to decide if there is a subset $A \subseteq S$ for which $|A| = k$ and

$$\sum_{w \in A} w = \underbrace{1 \cdots 1}_{\text{m times}} ,$$

where each sum is computed using bitwise-OR. For example, $1001 + 1010 = 1011$.