

Directions

- It's OK to solve both problems on the same sheet of paper.
- Make sure your name is on each sheet and that each problem part is properly labeled.

Problems

LO7. Do the following.

a. Let L denote the language of binary words that have either at least two 1's or at least 3 0's. Provide a succinct description for \bar{L} and provide words in this language.

Solution. All binary words with at most one 1 and at most two 0's, i.e.

$$\{\varepsilon, 0, 00, 10, 01, 100, 010, 001\}.$$

b. If A is the language consisting of words that have one 0, at least one 1, and an even number of 1's, while B is the language consisting of words having two 0's and an odd number of 1's, then is it true that $1110101101 \in AB$? Explain.

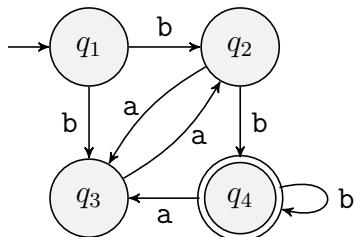
Solution. Yes, $11101 \in A$, while $01101 \in B$. Therefore, $1110101101 \in AB$.

c. Provide a regular expression that represents language A from part b.

Solution. We have

$$(11)^+0(11)^* \cup (11)^*0(11)^+ \cup 1(11)^*0(11)^*1.$$

LO8. Consider the NFA N shown below.



and let L denote the language that it recognizes.

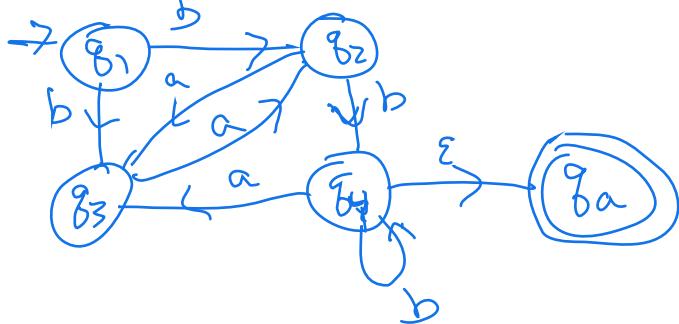
a. Use N to construct the NFA N' that accepts L^* and uses the algorithm described in lecture for this purpose.

Solution. Add a new initial state with an ε -edge from this state to the old initial state q_1 . Finally, add an ε -edge from q_4 to q_1 .

b. Demonstrate each step of the GNFA-to-Regular-Expression algorithm that computes a regular expression that describes L . Hint: your initial GNFA should have five states.

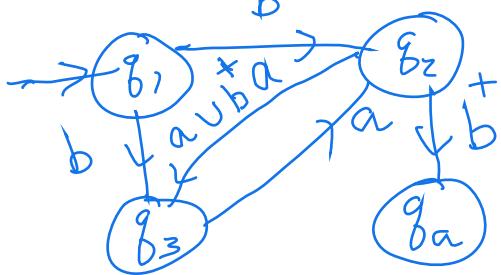
Solution.

1. Add new accepting state.



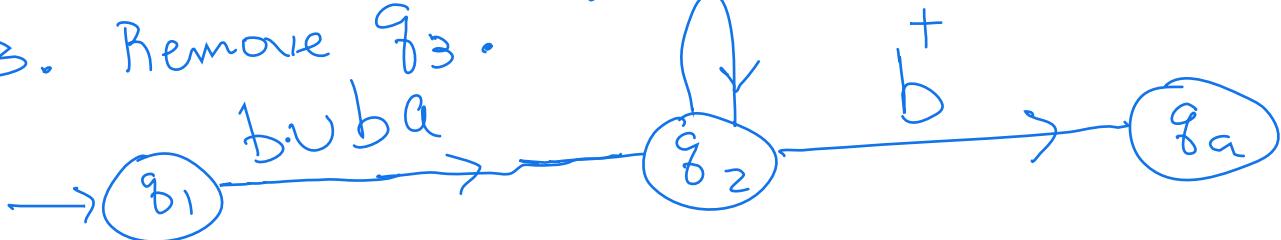
2. Remove q_4 .

$$\text{Note: } b^+ = bb^*$$

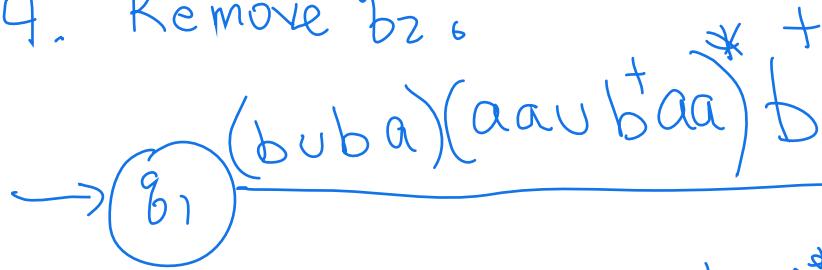


3. Remove q_3 .

$aabb^a$



4. Remove q_2 .



$(buba)(aabb^a)^*b$ describes $L(N)$.

Note: Solutions may vary based on the order of states removed.²