

## Directions

- It's OK to solve both problems on the same sheet of paper.
- Make sure your name is on each sheet and that each problem part is properly labeled.

## Problems

LO7. Do the following.

- a. Let  $L$  denote the language of binary words that have either at least two 1's or at least 3 0's. Provide a succinct description for  $\overline{L}$  and provide words in this language.

**Solution.** All binary words with at most one 1 and at most two 0's, i.e.

$$\{\varepsilon, 0, 00, 10, 01, 100, 010, 001\}.$$

- b. If  $A$  is the language consisting of words that have one 0, at least one 1, and an even number of 1's, while  $B$  is the language consisting of words having two 0's and an odd number of 1's, then is it true that  $1110101101 \in AB$ ? Explain.

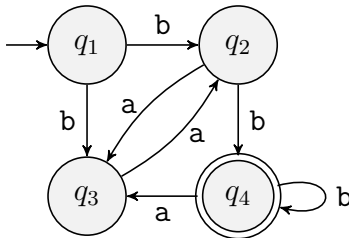
**Solution.** Yes,  $11101 \in A$ , while  $01101 \in B$ . Therefore,  $1110101101 \in AB$ .

- c. Provide a regular expression that represents language  $A$  from part b.

**Solution.** We have

$$(11)^+0(11)^* \cup (11)^*0(11)^+ \cup 1(11)^*0(11)^*1.$$

LO8. Consider the NFA  $N$  shown below.



and let  $L$  denote the language that it recognizes.

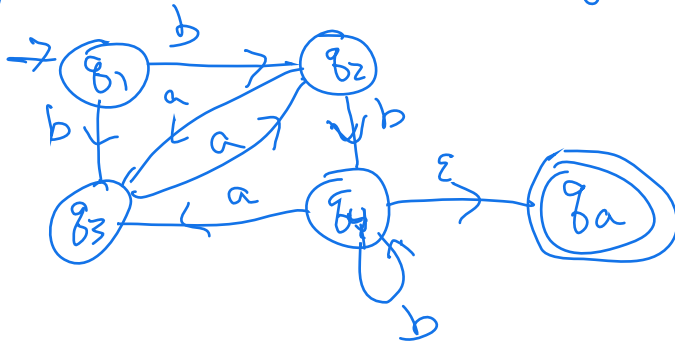
- a. Use  $N$  to construct the NFA  $N'$  that accepts  $L^*$  and uses the algorithm described in lecture for this purpose.

**Solution.** Add a new initial state with an  $\varepsilon$ -edge from this state to the old initial state  $q_1$ . Finally, add an  $\varepsilon$ -edge from  $q_4$  to  $q_1$ .

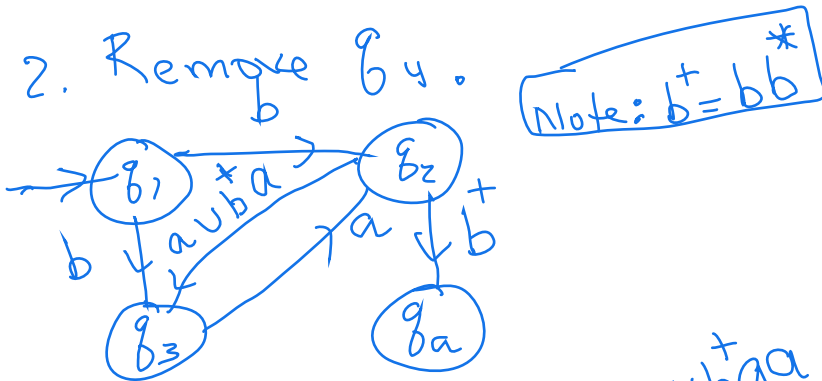
- b. Demonstrate each step of the GNFA-to-Regular-Expression algorithm that computes a regular expression that describes  $L$ . Hint: your initial GNFA should have five states.

**Solution.**

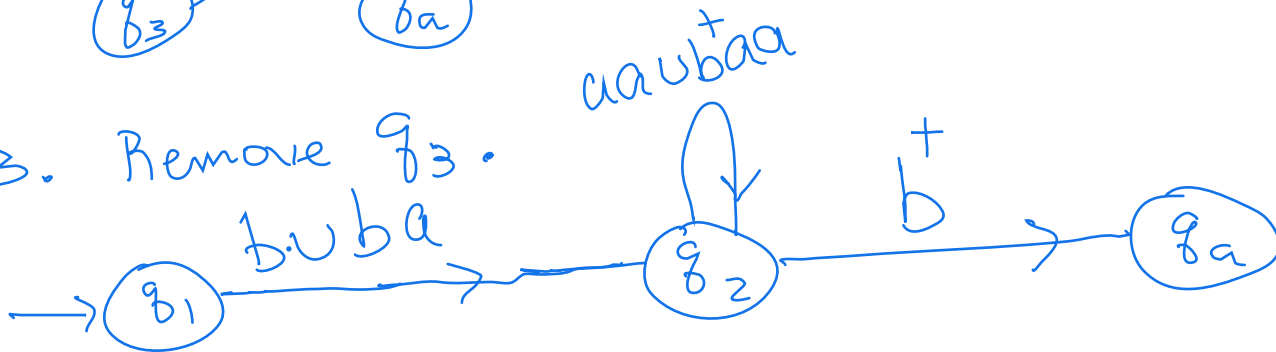
1. Add new accepting state.



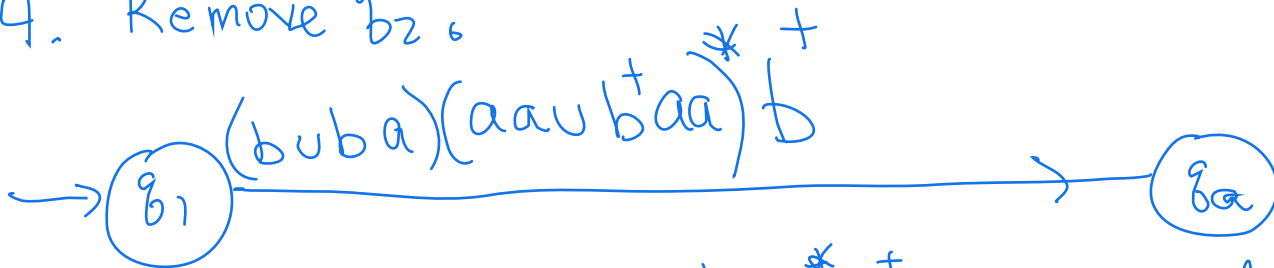
2. Remove  $q_4$ .



3. Remove  $q_3$ .



4. Remove  $q_2$ .



$(buba)(aaub^+aa)^+b$  describes  $L(N)$ .

Note: Solutions may vary based the order of states removed.