CECS 329, Quiz 3, Fall 2025, Dr. Ebert

IMPORTANT: READ THE FOLLOWING DIRECTIONS. Directions,

- For LO's 9 and 10 please solve each on a **SINGLE SEPARATE SHEET OF PAPER ONLY (BOTH FRONT AND BACK)**. Write **NAME** and **PROBLEM NUMBER** on each sheet.
- For LO's 1-8, it is OK to have multiple problems solved on the same sheet of paper.
- **THREE STACKS OF PAPERS: LO9, LO10, LO's 1-8**

Learning Outcome Assessment Problems

LO10. Do the following.

(a) The following is the δ-transition table for a Turing machine M. Show the first five configurations of M's computation on input word 10. Note: a is the initial state and e is the accepting state.

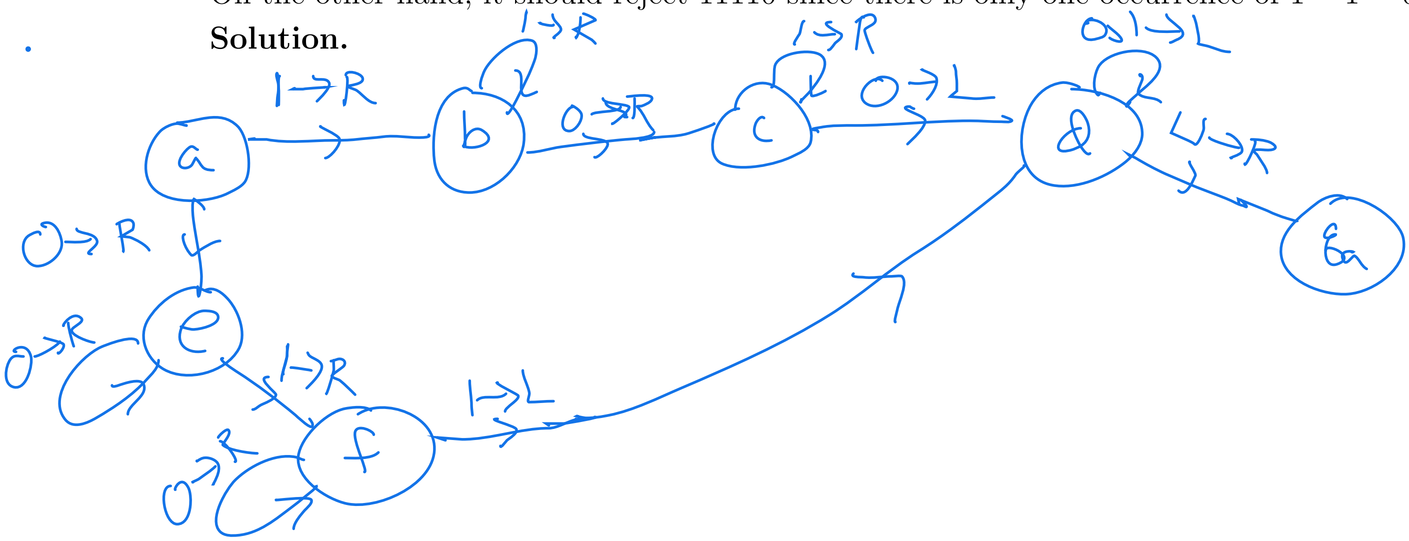
Q/Γ	0	1	$\sqcup$
a	b,$\sqcup$,R	c,$\sqcup$,R	n/a
b	b,0,R	c,0,R	d,$\sqcup$,L
c	b,1,R	c,1,R	d,$\sqcup$,L
d	d,0,L	d,1,L	e,$\sqcup$,L

Solution.

$$a10 \Rightarrow c0 \Rightarrow 1b\sqcup \Rightarrow d1 \Rightarrow d\sqcup 1.$$

(b) Provide the state diagram for a Turing machine M that has the following behavior. On (nonempty) binary input word w, M reads the first bit $b = w_1$ of w and proceeds to check if w has *at least two* occurrences of $1 - b$. If yes, then the head returns back to the first bit of w and enters the `accept` state. If no, then M immediately rejects w. For example, M should accept 0010110, since its first bit is 0 and there are three occurrences of $1 - 0 = 1$. On the other hand, it should reject 11110 since there is only one occurrence of $1 - 1 = 0$.

Solution.



LO9. Do the following.

(a) Recall the CFG $G = (V, \Sigma, R, E)$ from lecture for which We have $V = \{E, T, F\}$, $\Sigma = \{+, \times, a, b, (,)\}$, and R is the following set of rules.

$$E \to E + T \mid T$$

$$T \to T \times F \mid F$$

$$F \to (E) \mid a \mid b$$

Use G to provide a leftmost derivation of the arithmetic expression

$$a \times (a + b).$$

Solution. We have

$$E \to T \to T \times F \to F \times F \to a \times F$$

$$a \times (E) \to a \times (E + T) \to a \times (T + T) \to a \times (F + T) \to$$

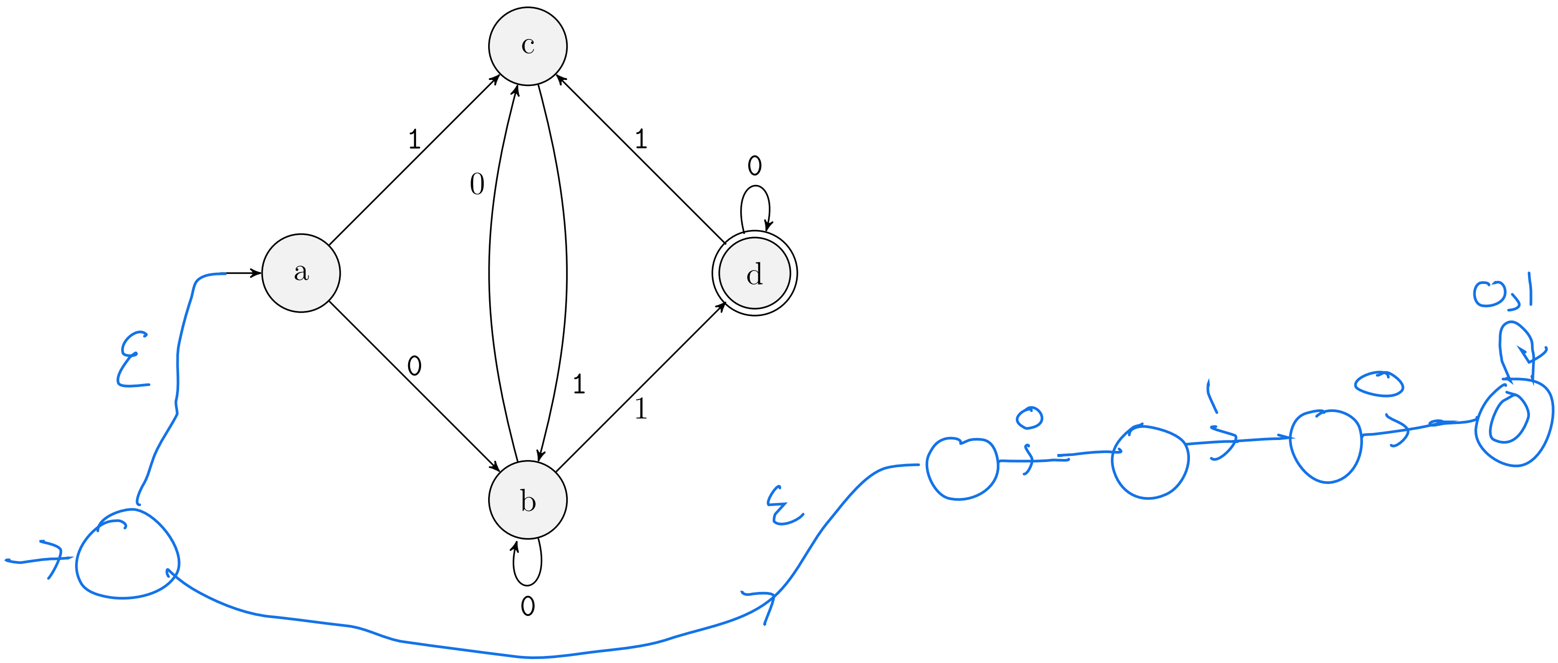
$$a \times (a + T) \to a \times (a + F) \to a \times (a + b).$$

(b) Let Σ be an alphabet and $s \in \Sigma$ be one of its letters. An s-palindrome is any word w over Σ for which, if $w_i = s$, then $w_{n-i+1} = s$, where $n = |w|$. For example, if $\Sigma = \{0, 1, 2\}$, then 0102020 is a 0-palindrome, since 0 appears at positions 1, $7-1+1 = 7$, 3, and $7-3+1 = 5$. One way of thinking about it is that a word is an s-palindrome iff, when replacing each non-s letter with some fixed letter, then the resulting word is a palindrome. For example, if we replace 1 and 2 with the number 3 in the word 0102020, then we get 0303030 which is a palindrome. Provide the rules of a context-free grammar that describes all words over the alphabet $\{0, 1, 2\}$ that are 0-palindromes and use it to derive the word 0102020.

Solution. We have the following rules with start variable S.

$$S \to 0S0 \mid 1S1 \mid 1S2 \mid 2S1 \mid 2S2 \mid 0 \mid 1 \mid 2 \mid \varepsilon.$$

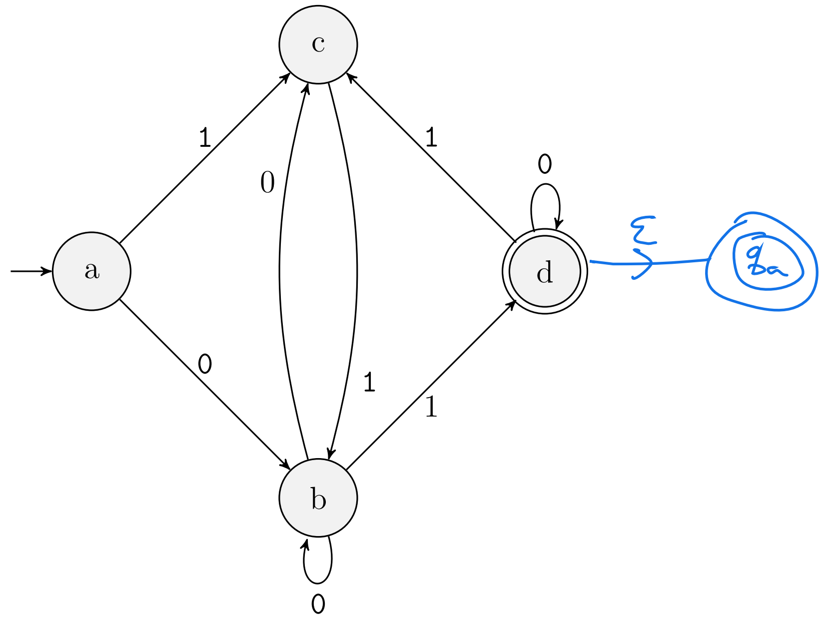
LO8. Consider the following NFA N.



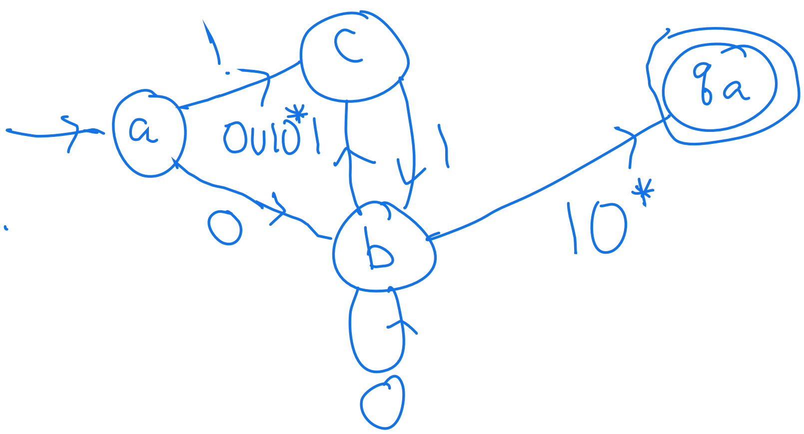
(a) If L_1 is the language accepted by N, then provide an NFA that accepts $L_1 \cup L_2$, where L_2 is the language consisting of all binary words that begin with 010.

Solution. See above.

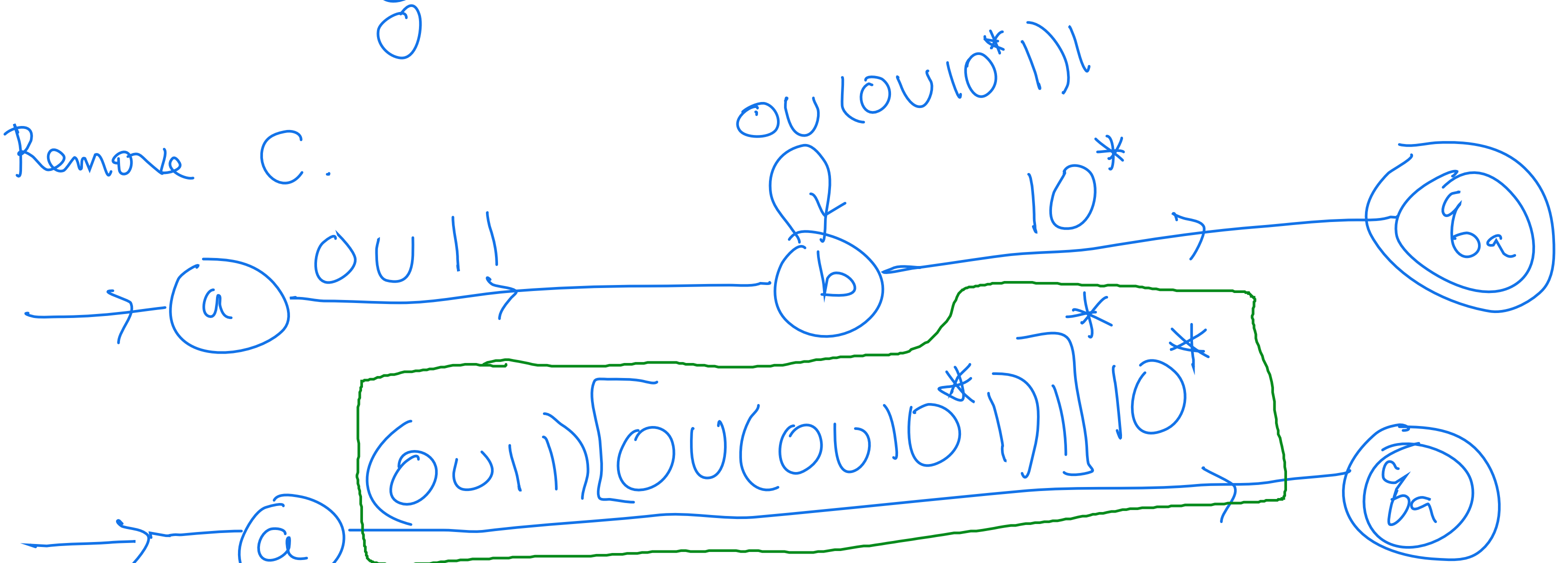
(b) Demonstrate each step of the GNFA-to-Regular-Expression algorithm that computes a regular expression that describes the language L_1 that is accepted by N. Hint: your first step should be to make a different accept state.



Solution. Remove d.



Remove C.



LO7. Do the following.

(a) Let A is the language that consists of all binary words that have an odd number of 0's, and B is the language that consists of all binary words that begin with 11, then describe the set of words that belong in $A \cap B$.

Solution. All binary words that begin with 11 and have an odd number of 0's.

(b) Does 11001001110110 belong to the language AB? Explain.

Solution. Yes, $11001001110 \in A$ and $110 \in B$.

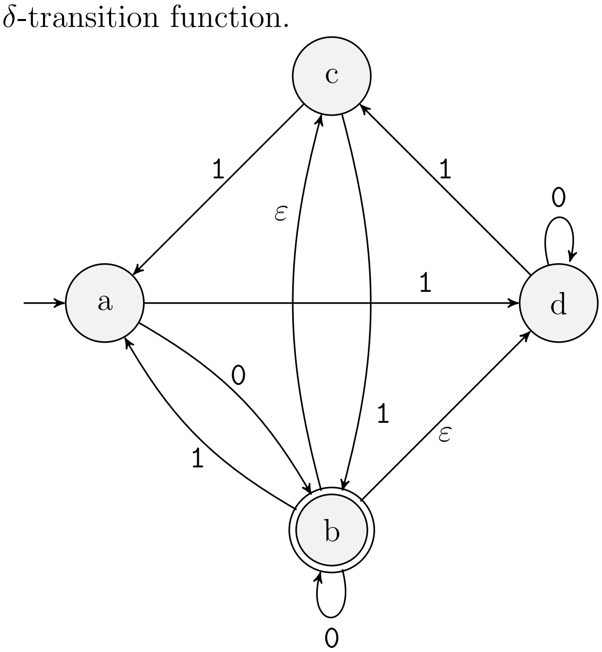
(c) Provide a regular expression that represents the language consisting of binary words that that contain an odd number of 0's or (inclusive) contain exactly two ones.

Solution.

$$0^*10^*10^* \cup 1^*0(1^*01^*01^*)1^*.$$

LO6. Do the following.

(a) For the NFA N whose state diagram is shown below, provide a table that represents N's δ-transition function.



(b) Use the table from part b to convert N to an equivalent DFA M using the method of subset states. Draw M's state diagram.

LO5. Do the following.

(a) Consider the language L of all binary words w that have at least two 0's and at most one 1. Provide a DFA M that accepts L.

(b) Demonstrate the computation of M on inputs i) $w_1 = 00100$ and ii) $w_2 = 00101$. For each computation, indicate whether w is accepted or rejected.

LO4. Answer the following.

(a) Consider the instance of `3SAT`

$$\mathcal{C} = \{c_1 = (\overline{x}_1, \overline{x}_3, x_5), c_2 = (x_2, \overline{x}_3, x_4), c_3 = (\overline{x}_2, \overline{x}_3, \overline{x}_5), c_4 = (x_1, x_3, \overline{x}_4), c_5 = (x_2, \overline{x}_4, x_5)\}.$$

and the mapping reduction f from `3SAT` to `DHP`. Answer the following with respect to $f(\mathcal{C}) = (G, a, b)$.

i. Consider the vertices lc_3 and rc_3 located in the x_4-diamond, as well as the c_3 clause vertex. Draw the edges that exist between these three vertices.

ii. Consider the c_1 clause vertex. Which diamonds do not have any vertices that are directly connected to c_1? Explain.

iii. Does G have a a DHP from a to b? If yes, then provide an itinerary for such a path that indicates i) the direction (left-to-right or right-to-left) to follow in each of the diamonds, and, for each clause, from which diamond will the clause be visited.

(b) Demonstrate the reduction from `Hamilton Cycle` to `TSP` presented in class and with respect to the `HC` instance G whose vertices and edges are listed in the LO2 problem below.

LO3. Answer the following.

(a) An instance of `Set Cover (SC)` is a triple $(\mathcal{S}, m, k)$, where $\mathcal{S} = \{S_1, \ldots, S_n\}$ is a collection of n subsets, where $S_i \subseteq \{1, \ldots, m\}$, for each $i = 1, \ldots, n$, and a nonnegative integer k. The problem is to decide if there are k subsets $S_{i_1}, \ldots, S_{i_k}$ for which

$$S_{i_1} \cup \cdots \cup S_{i_k} = \{1, \ldots, m\}.$$

An appropriate certificate for this problem is a subset $\mathcal{R} \subseteq S$ of S consisting of k-subsets from $\mathcal{S}$. Consider the following pseudocode for a verifier that, on inputs $(\mathcal{S}, m, k)$ and R, decides if there are k sets that cover all the numbers in $\{1, \ldots, m\}$. Note: a minimum of 18 points is needed to pass this LO.

Initalize array `covered` so that `covered`$[i] = 0$, for each $i = \{1, \ldots, m\}$.

For each $S \in \mathcal{R}$,

  For each $i \in S$,

    `covered`$[i] = 1$.

For each $i \in \{1, \ldots, m\}$,

  If `covered`$[i] == 0$, then return 0.

Return 1.

i. Describe the size parameters for `Set Cover`, including what each represents. Hint: there are two of them. (6 points)

ii. Provide the big-O number of steps that is required to execute the above psuedocode. (7 pts)

(b) Classify each of the following problems as being in `P`, `NP`, or co-`NP` (3 points each).

i. An instance of `Balanced Subset` is a set S of integers, and the problem is to decide if, for every member of S, it's additive inverse is also a member of S.

ii. An instance of `Boolean Subset Sum` is a set S of Boolean vectors of some common length, and a natural number $k \geq 0$, and the problem is to decide if there is a subset $A \subseteq S$ of size k for which the bitwise OR of the members of A is equal to 1.

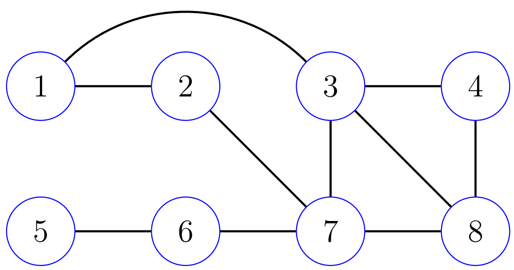
iii. An instance of `Long Increasing Subsequence` is a length-n array a of integers and a natural number $k \geq 0$, and the problem is to decide if there are indices $0 \leq i_1 \leq \cdots \leq i_k < n$ for which $a[i_1] \leq a[i_2] \leq \cdots \leq a[i_k]$.

iv. An instance of `Sum Avoidance` is a finite subset of integers S, and a target value t, and the problem is to decide if no subset of S has the property that its members sum to t.

LO2. Do the following.

(a) Provide the definition of what it means to be a mapping reduction from problem A to problem B. Hint: do *not* assume that A and B are decision problems. (6 pts)

(b) The simple graph $G = (V, E)$ shown below is an instance of the `Hamilton Path (HP)` decision problem. Provide $f(G)$, where f is the mapping reduction from `HP` to `LPath` provided in lecture. (7 pts)



(c) Verify that f is valid for input G from part b in the sense that both G and $f(G)$ have the same (decision) solution. Justify your answer.

LO1. Do/answer the following.

(a) Draw the implication graph $G_{\mathcal{C}}$ associated with the `2SAT` instance

$$\mathcal{C} = \{(x_1, x_3), (\overline{x}_1, \overline{x}_4), (x_2, x_5), (\overline{x}_2, \overline{x}_5), (\overline{x}_3, x_4), (\overline{x}_3, \overline{x}_4), (x_3, x_5)\}.$$

(b) Apply the improved `2SAT` algorithm to obtain a satisfying assignment for $\mathcal{C}$. When deciding on the next reachability set R_l to compute, follow the literal order $l = x_1, \overline{x}_1, \ldots, x_5, \overline{x}_5$. For each consistent reachability set encountered, provide the partial assignment α_{R_l} associated with R_l and draw the reduced implication graph before continuing to the next reachability set. Note: do *not* compute the reachability set for a literal that has already been assigned a truth value. Provide a final assignment α and verify that it satisfies all the clauses.

(c) Using the original `2SAT` algorithm, suppose `reachable`$(G_{\mathcal{C}}, x_2, \overline{x}_2)$ evaluates to 1. Then what must be true about any assignment α that satisfies $\mathcal{C}$? Explain.