

IMPORTANT: READ THE FOLLOWING DIRECTIONS. Directions,

- For LO's 9 and 10 please solve each on a **SINGLE SEPARATE SHEET OF PAPER ONLY (BOTH FRONT AND BACK)**. Write **NAME** and **PROBLEM NUMBER** on each sheet.
- For LO's 1-8, it is OK to have multiple problems solved on the same sheet of paper.
- **THREE STACKS OF PAPERS: LO9, LO10, LO's 1-8**

Learning Outcome Assessment Problems

LO10. Do the following.

- (a) The following is the δ -transition table for a Turing machine M . Show the first five configurations of M 's computation on input word 10. Note: a is the initial state and e is the accepting state.

Q/Γ	0	1	$\sqcup$
a	b, $\sqcup$,R	c, $\sqcup$,R	n/a
b	b,0,R	c,0,R	d, $\sqcup$,L
c	b,1,R	c,1,R	d, $\sqcup$,L
d	d,0,L	d,1,L	e, $\sqcup$,L

- (b) Provide the state diagram for a Turing machine M that has the following behavior. On (nonempty) binary input word w , M reads the first bit $b = w_1$ of w and proceeds to check if w has *at least two* occurrences of $1 - b$. If yes, then the head returns back to the first bit of w and enters the **accept** state. If no, then M immediately rejects w . For example, M should accept 0010110, since its first bit is 0 and there are three occurrences of $1 - 0 = 1$. On the other hand, it should reject 11110 since there is only one occurrence of $1 - 1 = 0$.

LO9. Do the following.

- (a) Recall the CFG $G = (V, \Sigma, R, E)$ from lecture for which We have $V = \{E, T, F\}$, $\Sigma = \{+, \times, a, b, (,)\}$, and R is the following set of rules.

$$E \rightarrow E + T \mid T$$

$$T \rightarrow T \times F \mid F$$

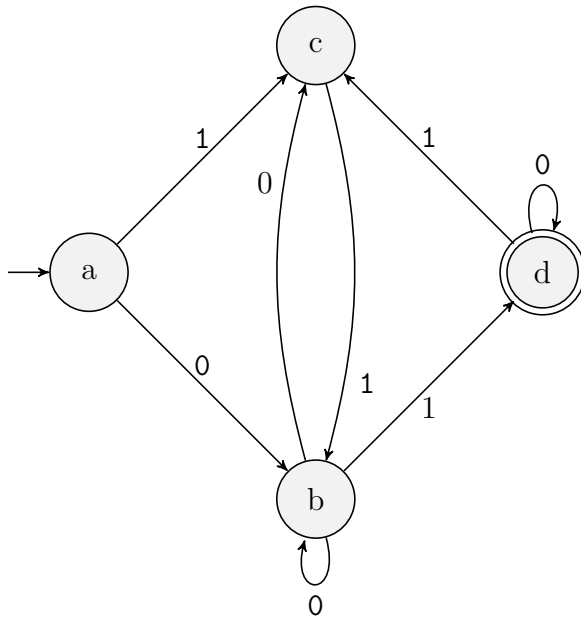
$$F \rightarrow (E) \mid a \mid b$$

Use G to provide a leftmost derivation of the arithmetic expression

$$a \times (a + b).$$

- (b) Let Σ be an alphabet and $s \in \Sigma$ be one of its letters. An s -palindrome is any word w over Σ for which, if $w_i = s$, then $w_{n-i+1} = s$, where $n = |w|$. For example, if $\Sigma = \{0, 1, 2\}$, then 0102020 is a 0-palindrome, since 0 appears at positions 1, $7-1+1 = 7$, 3, and $7-3+1 = 5$. One way of thinking about it is that a word is an s -palindrome iff, when replacing each non- s letter with some fixed letter, then the resulting word is a palindrome. For example, if we replace 1 and 2 with the number 3 in the word 0102020, then we get 0303030 which is a palindrome. Provide the rules of a context-free grammar that describes all words over the alphabet $\{0, 1, 2\}$ that are 0-palindromes and use it to derive the word 0102020.

LO8. Consider the following NFA N .



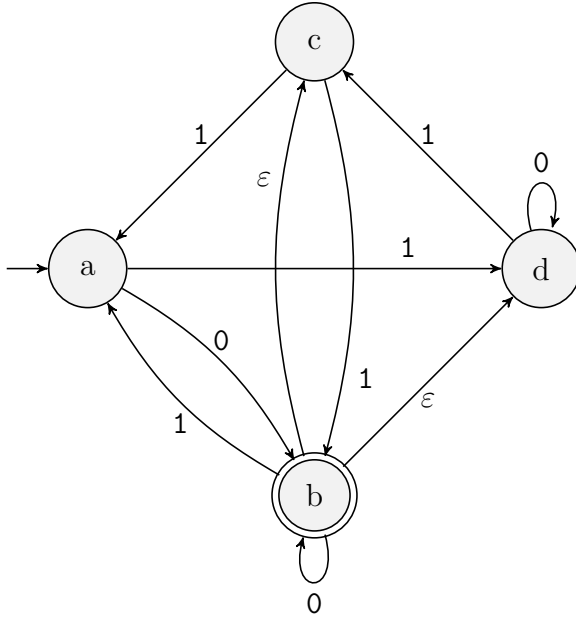
- (a) If L_1 is the language accepted by N , then provide an NFA that accepts $L_1 \cup L_2$, where L_2 is the language consisting of all binary words that begin with 010.
- (b) Demonstrate each step of the GNFA-to-Regular-Expression algorithm that computes a regular expression that describes the language L_1 that is accepted by N . Hint: your first step should be to make a different accept state.

LO7. Do the following.

- (a) Let A is the language that consists of all binary words that have an odd number of 0's, and B is the language that consists of all binary words that begin with 11, then describe the set of words that belong in $A \cap B$.
- (b) Does 11001001110110 belong to the language AB ? Explain.
- (c) Provide a regular expression that represents the language consisting of binary words that that contain an odd number of 0's or (inclusive) contain exactly two ones.

LO6. Do the following.

- (a) For the NFA N whose state diagram is shown below, provide a table that represents N 's δ -transition function.



- (b) Use the table from part b to convert N to an equivalent DFA M using the method of subset states. Draw M 's state diagram.

LO5. Do the following.

- Consider the language L of all binary words w that have at least two 0's and at most one 1. Provide a DFA M that accepts L .
- Demonstrate the computation of M on inputs i) $w_1 = 00100$ and ii) $w_2 = 00101$. For each computation, indicate whether w is accepted or rejected.

LO4. Answer the following.

- Consider the instance of 3SAT

$$\mathcal{C} = \{c_1 = (\bar{x}_1, \bar{x}_3, x_5), c_2 = (x_2, \bar{x}_3, x_4), c_3 = (\bar{x}_2, \bar{x}_3, \bar{x}_5), c_4 = (x_1, x_3, \bar{x}_4), c_5 = (x_2, \bar{x}_4, x_5)\}.$$

and the mapping reduction f from 3SAT to DHP. Answer the following with respect to $f(\mathcal{C}) = (G, a, b)$.

- Consider the vertices lc_3 and rc_3 located in the x_4 -diamond, as well as the c_3 clause vertex. Draw the edges that exist between these three vertices.
 - Consider the c_1 clause vertex. Which diamonds do not have any vertices that are directly connected to c_1 ? Explain.
 - Does G have a DHP from a to b ? If yes, then provide an itinerary for such a path that indicates i) the direction (left-to-right or right-to-left) to follow in each of the diamonds, and, for each clause, from which diamond will the clause be visited.
- Demonstrate the reduction from Hamilton Cycle to TSP presented in class and with respect to the HC instance G whose vertices and edges are listed in the LO2 problem below.

LO3. Answer the following.

- (a) An instance of **Set Cover (SC)** is a triple $(\mathcal{S}, m, k)$, where $\mathcal{S} = \{S_1, \dots, S_n\}$ is a collection of n subsets, where $S_i \subseteq \{1, \dots, m\}$, for each $i = 1, \dots, n$, and a nonnegative integer k . The problem is to decide if there are k subsets $S_{i_1}, \dots, S_{i_k}$ for which

$$S_{i_1} \cup \dots \cup S_{i_k} = \{1, \dots, m\}.$$

An appropriate certificate for this problem is a subset $\mathcal{R} \subseteq \mathcal{S}$ of $\mathcal{S}$ consisting of k -subsets from $\mathcal{S}$. Consider the following pseudocode for a verifier that, on inputs $(\mathcal{S}, m, k)$ and R , decides if there are k sets that cover all the numbers in $\{1, \dots, m\}$. Note: a minimum of 18 points is needed to pass this LO.

Initialize array **covered** so that **covered**[i] = 0, for each $i = \{1, \dots, m\}$.

For each $S \in \mathcal{R}$,

For each $i \in S$,

covered[i] = 1.

For each $i \in \{1, \dots, m\}$,

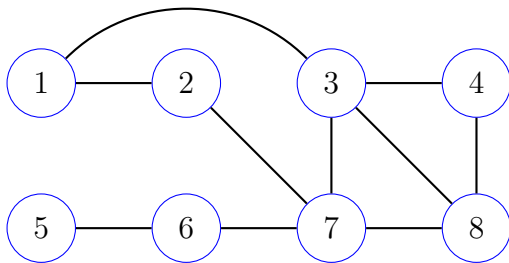
If **covered**[i] == 0, then return 0.

Return 1.

- i. Describe the size parameters for **Set Cover**, including what each represents. Hint: there are two of them. (6 points)
 - ii. Provide the big-O number of steps that is required to execute the above pseudocode. (7 pts)
- (b) Classify each of the following problems as being in P, NP, or co-NP (3 points each).
- i. An instance of **Balanced Subset** is a set S of integers, and the problem is to decide if, for every member of S , it's additive inverse is also a member of S .
 - ii. An instance of **Boolean Subset Sum** is a set S of Boolean vectors of some common length, and a natural number $k \geq 0$, and the problem is to decide if there is a subset $A \subseteq S$ of size k for which the bitwise OR of the members of A is equal to 1.
 - iii. An instance of **Long Increasing Subsequence** is a length- n array a of integers and a natural number $k \geq 0$, and the problem is to decide if there are indices $0 \leq i_1 \leq \dots \leq i_k < n$ for which $a[i_1] \leq a[i_2] \leq \dots \leq a[i_k]$.
 - iv. An instance of **Sum Avoidance** is a finite subset of integers S , and a target value t , and the problem is to decide if no subset of S has the property that its members sum to t .

LO2. Do the following.

- (a) Provide the definition of what it means to be a mapping reduction from problem A to problem B . Hint: do *not* assume that A and B are decision problems. (6 pts)
- (b) The simple graph $G = (V, E)$ shown below is an instance of the **Hamilton Path (HP)** decision problem. Provide $f(G)$, where f is the mapping reduction from HP to LPath provided in lecture. (7 pts)



- (c) Verify that f is valid for input G from part b in the sense that both G and $f(G)$ have the same (decision) solution. Justify your answer.

LO1. Do/answer the following.

- (a) Draw the implication graph $G_{\mathcal{C}}$ associated with the 2SAT instance

$$\mathcal{C} = \{(x_1, x_3), (\bar{x}_1, \bar{x}_4), (x_2, x_5), (\bar{x}_2, \bar{x}_5), (\bar{x}_3, x_4), (\bar{x}_3, \bar{x}_4), (x_3, x_5)\}.$$

- (b) Apply the improved 2SAT algorithm to obtain a satisfying assignment for $\mathcal{C}$. When deciding on the next reachability set R_l to compute, follow the literal order $l = x_1, \bar{x}_1, \dots, x_5, \bar{x}_5$. For each consistent reachability set encountered, provide the partial assignment α_{R_l} associated with R_l and draw the reduced implication graph before continuing to the next reachability set. Note: do *not* compute the reachability set for a literal that has already been assigned a truth value. Provide a final assignment α and verify that it satisfies all the clauses.
- (c) Using the original 2SAT algorithm, suppose $\text{reachable}(G_{\mathcal{C}}, x_2, \bar{x}_2)$ evaluates to 1. Then what must be true about any assignment α that satisfies $\mathcal{C}$? Explain.