CECS 528, Homework Assignment 4, Fall 2025, Dr. Ebert

Directions: Please review the `Homework` section on page 6 of the syllabus including a list of all rules and guidelines for writing and submitting solutions, This includes the statement regarding plagiarism.

Due Date: Saturday, December 13th as a PDF-file upload to the HW6 Canvas dropbox.

Problems

Note: a total point score of at least 50 is necessary for an LO11 pass. **Show all work for each problem solution**.

1. Do the following.

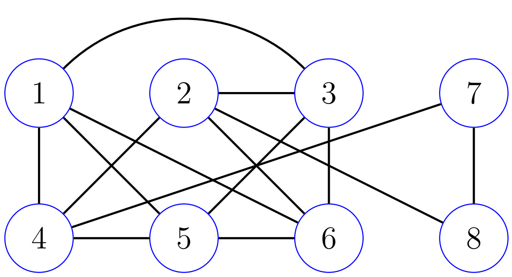
   (a) A bag consists of five coins. Three of the coins are two-headed, one is fair (one head and one tail), and the fifth is two-tailed. If three of the coins are randomly selected from the bag and are randomly flipped on to a table, determine the probability that at least two heads will appear. (8 pts)

   (b) For the experiment described in part a) determine the domain and probability distribution for the random variable $E[H|S]$, where H is the number of heads that appear on the table, and S is the number of two-headed coins that were selected. (7 pts)

   (c) Compute $E[H]$ in two different ways: i) directly by using only the probability distribution of H and ii) indirectly by using the probability distribution $E[H|S]$. (10 pts)

2. Suppose we apply Karger's algorithm to the graph shown below. Moreover, the edges selected in order are

$$(1,6),(2,3),(23,8),(238,4),(2348,5),(16,23458).$$



   (a) Show the resulting sequence of multigraphs that are formed after each edge selection (10 pts).

   (b) Provide the lower bound proved by Karger for the probability that the algorithm will return a minimum cut. (5 pts)

(c) Compute the actual probability of successfully finding the min cut $\{(4,7),(7,8)\}$ conditioned on the random choices of edges that were selected in each round. (10 pts)

3. Consider the problem of applying `WalkSAT` to a satisfiable instance of `2SAT` that has **four** variables and a unique satisfying assignment β. Letting F be the random variable that measures the number of bit flips that leads up to the event $\alpha = \beta$, our goal is to compute an upper-bound for $E[F|D = i]$, $i = 1, \dots, 4$, where D is a random variable that represents the current Hamming distance $d(\alpha, \beta)$. Note that $E[F|D = 0] = 0$.

   (a) Determine upper bounds for the values $E[F|D = 1], \dots, E[F|D = 4]$ by creating a system of recurrences, followed by solving the system. (15 pts)

   (b) Using your answers from part a, compute and upper bound for $E[F]$. Hint, use the Law of Expectation of Conditional Expectation (LECE). (10 pts)