CECS 528, Quiz 2, Fall 2025, Dr. Ebert

IMPORTANT: READ THE FOLLOWING DIRECTIONS. Directions,

- For each problem, write your solution using **ONE SHEET OF PAPER ONLY (BOTH FRONT AND BACK)**. Write **NAME** and **PROBLEM NUMBER** on each sheet.
- Write solutions to different problems on **SEPARATE SHEETS** of paper.
- It is OK to use the same sheet for parts of different make up problems.

Makeup LO Problems

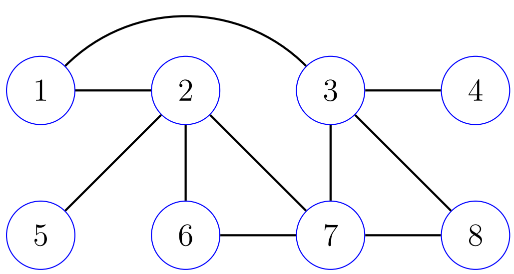
LO6. Suppose `2SAT` instance $\mathcal{C}$ has three variables and, when running the original `2SAT` algorithm the answer to each oracle query is shown in the table below. Is $\mathcal{C}$ satisfiable? If yes, provide a satisfying assignment for $\mathcal{C}$ and explain your reasoning. If not, again, explain your reasoning.

Oracle Query	**Answer**
`reachable`$(G_{\mathcal{C}}, x_1, \overline{x}_1)$	Yes
`reachable`$(G_{\mathcal{C}}, \overline{x}_1, x_1)$	No
`reachable`$(G_{\mathcal{C}}, x_2, \overline{x}_2)$	No
`reachable`$(G_{\mathcal{C}}, x_3, \overline{x}_3)$	Yes
`reachable`$(G_{\mathcal{C}}, \overline{x_3}, x_3)$	No

LO7. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B.

(b) For the mapping reduction f : `Vertex Cover` $\rightarrow$ `Half Vertex Cover`, draw $f(G, k)$ for the `Vertex Cover` instance whose graph is shown below, and for which $k = 3$.



(c) Verify that both (G, k) and $f(G, k)$ are either both positive instances, or are both negative ones. Explain and show work.

LO8. Answer the following.

(a) An instance of the `Complete Coloring` decision problem is a simple graph $G = (V, E)$ and a natural number $k \geq 1$. The problem is to decide if there is a mapping $f : V \to \{1, \ldots, k\}$ such that, for every pair of distinct numbers $i, j \in \{1, \ldots, k\}$, there is some edge $e = (u, v) \in E$ for which $f(u) = i$ and $f(v) = j$ (or equivalently, $f(u) = j$ and $f(v) = i$). Such a map is said to provide a **complete k-coloring** of G's vertices. Note: here we are thinking of the numbers $1, \ldots, k$ as representing k distinct colors. To see that `Complete Coloring` is an `NP` problem we define a certificate for instance (G, k) to be a mapping $f : V \to \{1, \ldots, k\}$. Provide pseudocode for a verifier that, on inputs (G, k) and $f : V \to \{1, \ldots, k\}$, decides if f is a complete k-coloring of the vertices of G. (7 pts) Note: a minimum of 18 points is needed to pass this LO.

(b) Given that $m = |E|$ and $n = |V|$ are the size parameters for the `Complete Coloring` problem and $k \leq n$, provide the big-O number of steps that is required to execute your psuedocode. Your analysis should include any hidden costs (for example, the number of steps needed to check if a pair of vertices represents an edge in a graph depends on the implementation and we cannot just assume it is O(1)). (6 pts)

(c) Classify each of the following problems as being in `P`, `NP`, or co-`NP` (3 points each).

i. An instance of `Boolean Subset Sum` is a set S of Boolean vectors of some common length, and the problem is to decide if, for each subset $A \subseteq S$, the bitwise OR of the members of A is equal to 1.

ii. An instance of the `LPath` decision problem is a pair (G, k), where $G = (V, E)$ is a simple graph, and $k \geq 0$ is a nonnegative integer. The problem is to decide if G has a simple path of length k.

iii. An instance of `Quadratic Diophantine` is a triple (a, b, c) of positive integers, and the problem is to decide if there is a pair of positive integers x and y for which $ax^2 + by = c$.

iv. An instance of `Moon Cycle` is a simple graph $G = (V, E)$ and the problem is to decide if G has a cycle of length of either 29 or 30.