CECS 528, Quiz 3, Fall 2025, Dr. Ebert

IMPORTANT: READ THE FOLLOWING DIRECTIONS. Directions,

- For LO's 9 and 10 please solve each on a **SINGLE SEPARATE SHEET OF PAPER ONLY (BOTH FRONT AND BACK)**. Write **NAME** and **PROBLEM NUMBER** on each sheet.
- For LO's 1-8, it is OK to have multiple problems solved on the same sheet of paper.
- **THEREFORE, THREE STACKS OF PAPERS: LO9, LO10, LO's 1-8**

Makeup LO Problems

LO10. Answer/Do the following.

(a) Recall the approximation algorithm for `Load Balancing`. Suppose that, upon completion of the algorithm, processor P_1 finishes the latest at M units of time and let t be the duration of the last (small) task that was added to P_1. Why is it true that, when this task was assigned to P_1, that every other processor had a duration that was at least $M - t$? What is the bound on the value of t? Explain.

(b) Demonstrate the approximation algorithm for `LB` using the following tasks with $p = 3$ processors and $\eta = 1/11$.

Task	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}
Duration	7	4	7	1	1	6	7	4	6	6	3	1	6	1	1	5

LO9. Answer the following.

(a) Consider the instance of `3SAT`

$$\mathcal{C} = \{c_1 = (\overline{x}_1, \overline{x}_3, x_5), c_2 = (x_2, \overline{x}_3, x_4), c_3 = (\overline{x}_2, \overline{x}_3, \overline{x}_5), c_4 = (x_1, x_3, \overline{x}_4), c_5 = (x_2, \overline{x}_4, x_5)\}.$$

and the mapping reduction f from `3SAT` to `DHP`. Answer the following with respect to $f(\mathcal{C}) = (G, a, b)$.

i. Consider the vertices lc_3 and rc_3 located in the x_4-diamond, as well as the c_3 clause vertex. Draw the edges that exist between these three vertices.

ii. Consider the c_1 clause vertex. Which diamonds do not have any vertices that are directly connected to c_1? Explain.

iii. Does G have a a DHP from a to b? If yes, then provide an itinerary for such a path that indicates i) the direction (left-to-right or right-to-left) to follow in each of the diamonds, and, for each clause, from which diamond will the clause be visited.

(b) Demonstrate the reduction from `Hamilton Cycle` to `TSP` presented in class and with respect to the `HC` instance G whose vertices and edges are listed in the LO7 problem below.

LO8. Answer the following.

(a) An instance of `Set Cover (SC)` is a triple $(\mathcal{S}, m, k)$, where $\mathcal{S} = \{S_1, \ldots, S_n\}$ is a collection of n subsets, where $S_i \subseteq \{1, \ldots, m\}$, for each $i = 1, \ldots, n$, and a nonnegative integer k. The problem is to decide if there are k subsets $S_{i_1}, \ldots, S_{i_k}$ for which

$$S_{i_1} \cup \cdots \cup S_{i_k} = \{1, \ldots, m\}.$$

An appropriate certificate for this problem is a subset $R \subseteq S$ of S consisting of k-subsets: $R = \{S_{i_1}, \ldots, S_{i_k}\}$. Provide pseudocode for a verifier that, on inputs $(\mathcal{S}, m, k)$ and R, decides if there are k sets that cover all the numbers in $\{1, \ldots, m\}$. (7 pts) Note: a minimum of 18 points is needed to pass this LO.

i. For a given instance $(\mathcal{S}, m, k)$ of `SC` describe a certificate in relation to $(\mathcal{S}, m, k)$.

ii. Provide a semi-formal verifier algorithm that takes as input i) an instance $(\mathcal{S}, m, k)$, and ii) a certificate for $(\mathcal{S}, m, k)$ as defined in part a, and decides if the certificate is valid for $(\mathcal{S}, m, k)$.

iii. Given that m and $n = |S|$ are the size parameters for the `Set Cover` problem, provide the big-O number of steps that is required to execute your psuedocode. Your analysis should include any hidden costs. (6 pts)

(b) Classify each of the following problems as being in `P`, `NP`, or co-`NP` (3 points each).

i. An instance of `Balanced Subset` is a set S of integers, and the problem is to decide if, for every member of S, it's additive inverse is also a member of S.

ii. An instance of `Boolean Subset Sum` is a set S of Boolean vectors of some common length, and a natural number $k \geq 0$, and the problem is to decide if there is a subset $A \subseteq S$ of size k for which the bitwise OR of the members of A is equal to 1.

iii. An instance of `Long Increasing Subsequence` is a length-n array a of integers and a natural number $k \geq 0$, and the problem is to decide if there are indices $0 \leq i_1 \leq \cdots \leq i_k < n$ for which $a[i_1] \leq a[i_2] \leq \cdots \leq a[i_k]$.

iv. An instance of `Sum Avoidance` is a finite subset of integers S, and a target value t, and the problem is to decide if no subset of S has the property that its members sum to t.

LO7. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B.

(b) For the mapping reduction f : `Maximum Bipartite Matching` $\rightarrow$ `Maximum Flow`, draw $f(G)$ for `MBM` instance $G = (U, V, E)$, where $U = \{u_1, u_2, u_3, u_4\}$, $V = \{v_1, v_2, v_3, v_4\}$, and

$$E = \{(u_1, v_1), (u_1, v_2), (u_1, v_4), (u_2, v_1), (u_2, v_3),$$

$$(u_2, v_4), (u_3, v_1), (u_3, v_3), (u_4, v_1), (u_4, v_3)\}.$$

(c) Verify that both G and $f(G)$ have the same solution, where we assume that a "solution" to each problem instance is a nonnegative integer. Defend your answer by providing details of each solution.

LO6. Do/answer the following.

(a) Draw the implication graph $G_{\mathcal{C}}$ associated with the `2SAT` instance

$$\mathcal{C} = \{(x_1, x_3), (\overline{x}_1, \overline{x}_4), (x_2, x_5), (\overline{x}_2, \overline{x}_5), (\overline{x}_3, x_4), (\overline{x}_3, \overline{x}_4), (x_3, x_5)\}.$$

(b) Apply the improved `2SAT` algorithm to obtain a satisfying assignment for $\mathcal{C}$. When deciding on the next reachability set R_l to compute, follow the literal order $l = x_1, \overline{x}_1, \ldots, x_5, \overline{x}_5$. For each consistent reachability set encountered, provide the partial assignment α_{R_l} associated with R_l and draw the reduced implication graph before continuing to the next reachability set. Note: do *not* compute the reachability set for a literal that has already been assigned a truth value. Provide a final assignment α and verify that it satisfies all the clauses.

(c) Using the original `2SAT` algorithm, suppose `reachable`$(G_{\mathcal{C}}, x_2, \overline{x}_2)$ evaluates to 1. Then what must be true about any assignment α that satisfies $\mathcal{C}$? Explain.

LO5. Solve the following problems.

(a) The dynamic-programming algorithm that solves the `Longest Common Subsequence (LCS)` optimization problem defines a recurrence for the function $\text{lcs}(i, j)$. In words, what does $\text{lcs}(i, j)$ equal? Hint: do *not* write the recurrence (see Part b).

(b) Provide the dynamic-programming recurrence for $\text{lcs}(i, j)$.

(c) Apply the recurrence from Part b to the words $u =$ baabab and $v =$ bbbaaa. Show the matrix of subproblem solutions and use it to provide an optimal solution.

LO4. Do the following.

(a) For the weighted graph with edges

$$(a, e, 6), (b, e, 4), (c, e, 3), (c, d, 5), (d, f, 2), (e, f, 1),$$

Show how the disjoint-set data structure forest changes when processing each edge in Kruskal's sorted list of edges. When unioning two trees, use the convention that the root of the union is the root which has the *lower* alphabetical order. For example, if two trees, one with root a, the other with root b, are to be unioned, then the unioned tree should have root a.

(b) State the greedy choice that is being made in each round of Prim's algorithm. The weighted edges of a graph $G = (V, E)$ are

$$E = \{(1, 2, 11), (1, 3, 19), (1, 4, 15), (1, 5, 13), (1, 6, 8), (2, 3, 14), (2, 4, 17),$$

$$(2, 5, 16), (2, 6, 22), (3, 4, 10), (3, 5, 12), (3, 6, 19), (4, 5, 15), (5, 6, 20)\}.$$

For each round of Prim's algorithm applied to G, indicate the selection for that round and provide a drawing of the final output of the algorithm. Hint: you do *not* need to use a heap data structure. Also, break any ties by choosing the vertex having least value. For example, if there is a tie beetween vertices 2 and 4, then choose vertex 2.

LO3. Do the following.

(a) Consider the FFT algorithm when applied to a polynomial $A(x)$ having degree $2^n - 1$. Provide the equation that relates $A(x)$ to the two subproblem polynomials $A_e(x)$ and $A_o(x)$. What are the degrees of these two polynomials? Based on this equation, why is it essential that, for even n, the nth roots of unity come in additive-inverse pairs?

(b) If $p(x) = -2 + 3x + x^2 - 5x^3$, then compute $\text{DFT}^{-1}(p)$ using the FFT algorithm. Show the entire recursion tree as was done in the lecture notes. (10 points)

LO2. Solve each of the following problems.

(a) Recall that the `Find-Statistic` algorithm makes use of the Partitioning algorithm and uses a pivot that is guaranteed to have at least

$$3(\lfloor\frac{1}{2}\lceil\frac{n}{5}\rceil\rfloor - 2) \geq 3(\frac{1}{2}\cdot\frac{n}{5} - 3) = \frac{3n}{10} - 9.$$

members of array a both to its left and to its right. Explain the rationale behind the following numbers that appear on the left side of the inequality: i) 3, ii) $\frac{1}{2}$, and iii) -2.

(b) Demonstrate the partitioning step of Hoare's version of `Quicksort` for the array

$$a = 4, 8, 1, 6, 2, 7, 5, 3, 9, 11, 10$$

where we assume that the pivot equals the median of the first, last, and middle integers of a.

LO1. Solve the following problems.

(a) Use the Master Theorem to determine the growth of $T(n)$ if it satisfies the recurrence $T(n) = 9T(n/3) + n^{\log_4 16} \log^3 n$. Defend your answer.

(b) Use the substitution method to prove that, if $T(n)$ satisfies

$$T(n) = 4T(n/2) + n \log n$$

then $T(n) = \Omega(n^2)$.