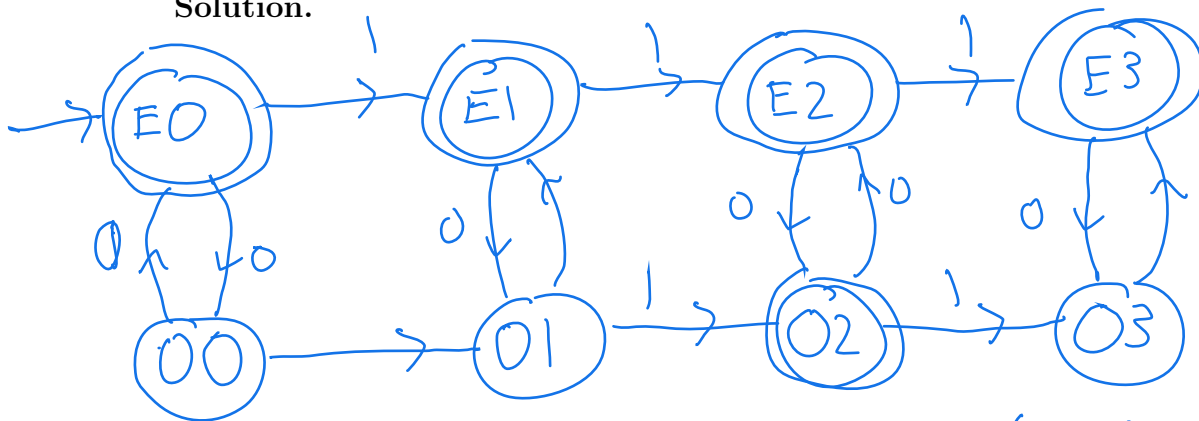


IMPORTANT: For each problem (LO1, LO2, GK, Advanced, All LO Makeups) write your solution using a **SINGLE** and **SEPARATE SHEET OF PAPER (BOTH FRONT AND BACK)**. Write your **FIRST NAME, ALPHABETIZING NAME** and **PROBLEM NUMBER** on each sheet.

25 Points Each

- LO1** a. Provide the state diagram for a DFA M that accepts exactly those words that contain an even number of 0's or (inclusive) contain exactly two ones. For example, 00100, 01010, and 1001 should all be accepted, but not 100101. (19 pts)

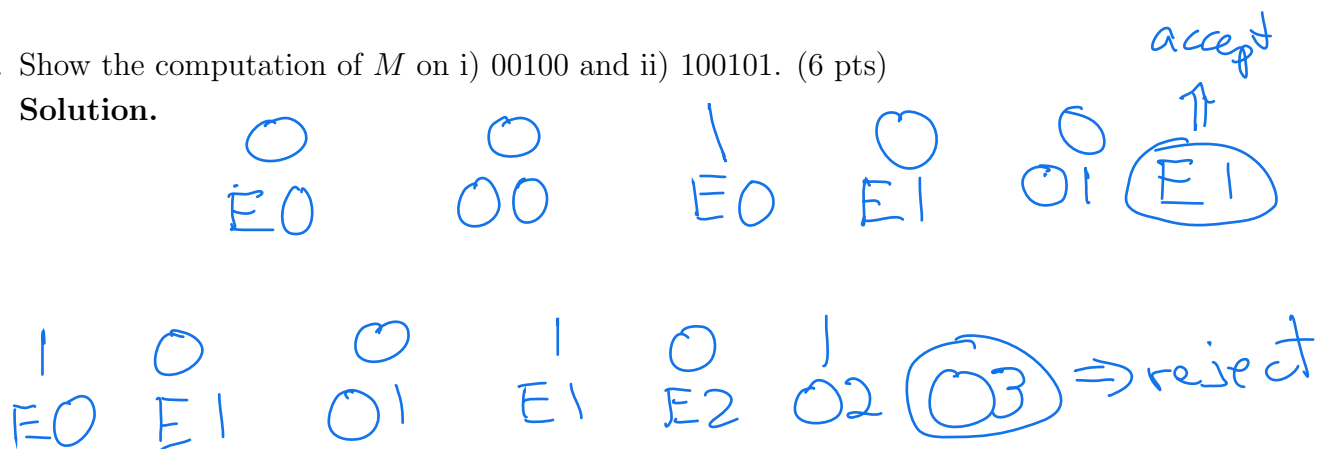
Solution.



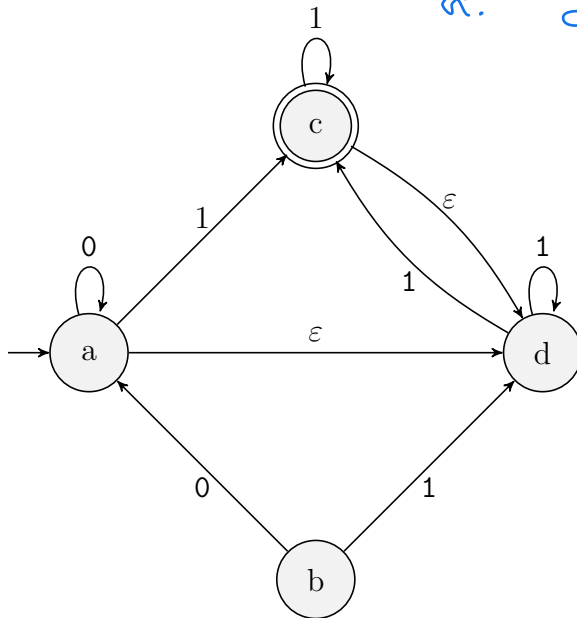
E_i : "even # of 0's and i 1's (or $i \geq 3$ in case of E_3)"
 O_i : "odd # of 0's and i 1's"

- b. Show the computation of M on i) 00100 and ii) 100101. (6 pts)

Solution.



LO2 Do the following for the NFA N whose state diagram is shown below.

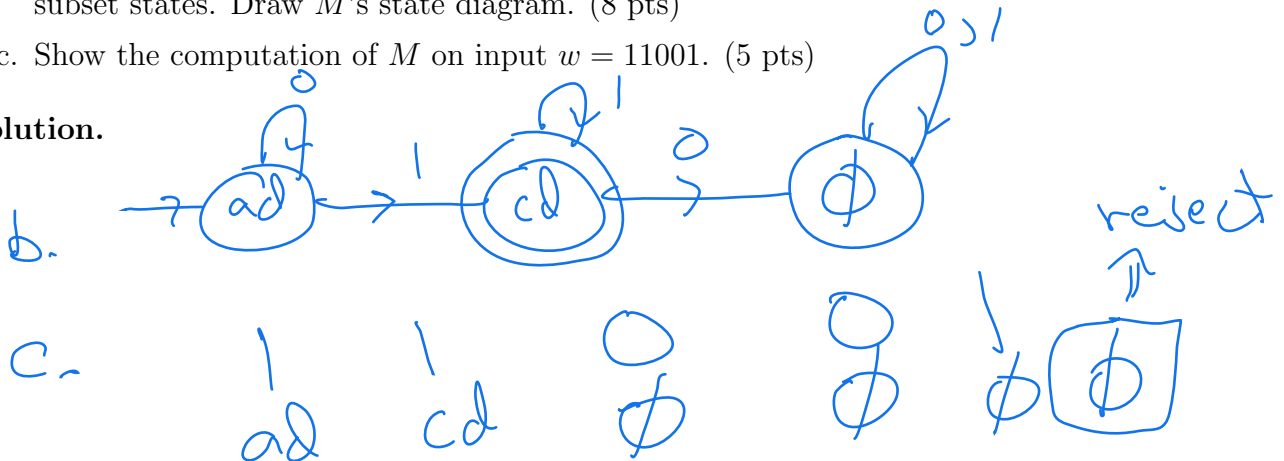


a. δ

$Q \backslash \Sigma$	0	1
a	ad	cd
b	ad	d
c	$\emptyset$	cd
d	$\emptyset$	cd

- Provide a table that represents N 's δ transition function. (12 pts)
- Use the table from part a to convert N to an equivalent DFA M using the method of subset states. Draw M 's state diagram. (8 pts)
- Show the computation of M on input $w = 11001$. (5 pts)

Solution.



- GK** a. For the δ -transition function of a DFA M what are its inputs and what does it output (within the context of a computation)? How does an NFA's δ -transition function differ from that of a DFA's? (15 pts)

Solution. The signature of the δ -transition function is $\delta : Q \times \Sigma \rightarrow Q$. Thus, δ has two inputs: the current state q of the computation and the current symbol $s \in \Sigma$ being read. It then outputs $\delta(q, s) = q_{\text{next}}$ which gives the next state of the computation. An NFA's δ -transition function differs from that in a DFA in that it outputs a subset of states which allows the NFA to simultaneously transition to multiple states.

- b. Which of the following (there is only one) does *not* describe a regular language: i) all binary words that begin and end with the same number of 0's. ii) all binary words having a number of 1's that is divisible by 5, iii) all binary words that have the pattern 011001, iv) all binary words except for the word 1101. Defend your choice. (10 pts)

Solution. Language i) is the only one that is not regular. This is so because the machine would have to keep track of the number of 0's at the beginning of the word and match that number with the number of 0's at the end of the word. But the number of possible 0's is unlimited and so the amount of memory needed is also unlimited which contradicts the fact that DFA's only have a finite amount of memory in the form of states.

ADV Consider the alphabet

$$\Sigma = \left\{ \begin{array}{cccc} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{array} \right\}$$

where each symbol is a column vector of two bits. Provide the state diagram for a DFA M that accepts all words w over Σ for which the top and bottom layers of w represent binary words u and v , respectively, where v is equal to u shifted two bits to the right, with the first two bits of v filled in with 0's. Notice that, because of the shift, the last two bits of u will not appear in v . For example, the DFA should accept

$$w_1 = \begin{array}{r} 010101 \\ 000101 \end{array}$$

However, it should reject

$$w_2 = \begin{array}{r} 10001 \\ 10100 \end{array}$$

since the first bit of v should equal 0. Provide a description of each state of M . In other words, for each state $q \in Q$, what will be true about a word that forces M to terminate in state q ? (25 pts)

Solution.

State ij : The next two bits that v should have are i & j where $i, j \in \{0, 1\}$
 State R : reject.

