CECS 528, LO Assessment 1 Solutions, Fall 2026, Dr. Ebert

IMPORTANT: For each problem (LO1,LO2,GK,Advanced, All LO Makeups) write your solution using a **SINGLE** and **SEPARATE SHEET OF PAPER (FRONT AND BACK)**. Write your **FIRST NAME**, **ALPHABETIZING NAME** and **PROBLEM NUMBER** on each sheet.

25 Points Each

LO1 a. Use the Master Theorem to determine the growth of $T(n)$ if it satisfies the recurrence $T(n) = 9T(n/3) + 1000n^2 + n$. Defend your answer. (10 pts)

Solution. $f(n) = 1000n^2 + n = \Theta(n^2)$. Similarly $n^{\log_3 9} = n^2 = \Theta(n^2)$. Thus, by Case 2 of MT, $T(n) = \Theta(n^2 \log n)$.

b. Use the Substitution method to establish that, if $T(n) = T(n/5) + T(n/4) + n$, then $T(n) = \Theta(n)$. Note: this proves that the `Find Statistic` algorithm requires $O(n)$ steps. (15 pts)

Solution. Inductive assumption: $T(k) \leq Ck$ for all $k < n$ and some constant $C > 0$. Prove that $T(n) \leq Cn$. We have

$$T(n) = T(n/5) + T(n/4) + n \leq Cn/5 + Cn/4 + n = 9Cn/20 + n \leq Cn \Leftrightarrow$$

$$11C/20 \geq 1 \Leftrightarrow C \geq \frac{20}{11}.$$

LO2 a. Demonstrate the `Median-of-Five Find Statistic Algorithm` when applied to the array

$$a = 86, 96, 35, 16, 36, 94, 55, 50, 77, 8, 44, 15, 19, 32, 55, 98, 68, 78, 70, 7, 3, 20, 87$$

with $k = 7$, lower $= 0$, and upper $= 22$. Compute the pivot M, the arrays a_{left} and a_{right}, and the updated lower and upper values. Hint: although you do not need to show the step by step details, nevertheless you should be applying the partitioning algorithm from Hoare's `Quicksort`.

Solution. $M = \text{median}(36, 55, 32, 70, 20) = 36$. After the partitioning step we have

$$a_{\text{left}} = 20, 3, 35, 16, 7, 32, 19, 15, 8$$

and

$$a_{\text{right}} = 44, 50, 55, 94, 55, 98, 68, 78, 70, 87, 96, 86, 77.$$

b. What is its worst-case running time of Hoare's `Quicksort` and what must happen in order for it to occur? Explain and defend your answer using an arithmetic series and evaluating the growth of that series. (13 pts)

Solution. See Divide and Conquer lecture notes.

GK a. Using just a few sentences, why is the Master Theorem by itself insufficient for determining the big-O number of steps that is needed to execute any arbitrary divide-and-conquer algorithm? (10 pts)

Solution. Some divide-and-conquer algorithms, such as `Find Statistic`, yield non-uniform recurrences for which the Master Theorem does not apply.

b. For a uniform divide-and-conquer recurrence, what do the a, b, and $f(n)$ represent assuming that $T(n)$ is describing the number of steps that are needed to execute some divide-and-conquer algorithm?

Solution. See Divide and Conquer lecture notes.

ADV a. Consider an array $a = (x_1, y_1), \ldots, (x_n, y_n)$ of two-dimensional points where each component is an integer. Describe in a paragraph an $O(n)$ algorithm that takes a as input along with a value k, $0 \leq k < n$, and returns the k points of a that are furthest from the origin $(0, 0)$, where distance is calculated using the Euclidean distance:

$$\sqrt{x_1^2 + y_1^2}.$$

Hint: you may use any of the algorithms that were covered presented in the Divide and Conquer lecture. (10 points)

Solution. Create an array b for which $b[i] = x_i^2 + y_i^2$, for each $1 \leq i \leq n$. Note: in reality $b[i]$ would be a data structure whose attributes are i) the original point ii) the square of the distance of that point from the origin. Now call `Find-Statistic` on inputs b, $k' = n - k + 1$, `lower` $= 1$, and `upper` $= n$. After doing so, the final k members of b will be the furthest away from the origin.

b. Convert your paragraph description to rigorous pseudocode. Make sure to clearly define all variables and all functions that are being called. (15 pts)

Solution.

Input: $a = (x_1, y_1), \ldots, (x_n, y_n)$, where $n \geq k$.

Output: the k points of a that are furthest from the origin.

For each $i \in \{1, \ldots n\}$,

  $b[i] = x_i^2 + y_i^2$.

find_statistic$(b, n - k + 1, 1, n)$.

Return $b[n - k + 1 : n]$. //Return the final k numbers of b as an array

If $u = v$, then return `true`.