

Example 2 of Recurrence Relations Lecture

Example 2 of Recurrence Lecture. Determine the order of growth of $T(n)$ for the following recurrences.

1. $T(n) = 16T(n/4) + n$

$n^{\log_4 16} = n^2$

$f(n) = n = O(n^{2-\epsilon})$, where ϵ is any positive number ≤ 1 .

∴ By Case 1 of M.T., $T(n) = \Theta(n^2)$

2. $T(n) = T(n/5) + 20$

$n^{\log_5 1} = n^0 = 1$

$f(n) = 20 = \Theta(1)$

∴ By Case 2 of M.T., $T(n) = \Theta(1 \cdot \log n) = \Theta(\log n)$

$\log_4 3 < 1$

3. $T(n) = 3T(n/4) + n \log n$

$n^{\log_4 3}$

$f(n) = \Omega(n^{\log_4 3 + \epsilon})$ where $\log_4 3 + \epsilon \leq 1 \Rightarrow \epsilon \leq 1 - \log_4 3$

∴ By Case 3 of M.T. $T(n) = \Theta(n \log n)$

$$4. \quad T(n) = \underbrace{2}_{a} T(\underbrace{n/2}_{b}) + \underbrace{n \log n}_{f(n)}$$

$$n^{\log_2 2} = n^1$$

$$f(n) = n^1 \cdot \log n$$

∴ By case 4 of M.T.

$$T(n) = \Theta(n \log^2 n)$$