

NO NOTES, BOOKS, ELECTRONIC DEVICES, OR INTERPERSONAL COMMUNICATION ALLOWED. Submit each solution on a separate sheet of paper.

Problem

LO1. Determine the asymptotic growth of the sum

$$\frac{\ln 1}{1} + \frac{\ln 2}{2} + \dots + \frac{\ln n}{n} = \Theta\left(\int_1^n \frac{\ln x}{x} dx\right)$$

Show all work and justify your approach.

$$\begin{aligned} u = \ln x \quad du &= \frac{1}{x} dx \quad \Rightarrow \\ \Theta\left(\int_0^{\ln n} u du\right) &= \Theta\left(\frac{1}{2} u^2 \Big|_0^{\ln n}\right) = \\ &= \Theta\left(\frac{1}{2} \ln^2 n\right) = \Theta(\ln^2 n) \\ &= \Theta(\log^2 n) \end{aligned}$$