CECS 528, Learning Outcome Assessment 10a, April 21st, Spring 2023, Dr. Ebert

Problems

LO6. Recall that the `find_statistic` algorithm makes use of Quicksort's partitioning algorithm and uses a pivot that is guaranteed to have at least

$$3(\lfloor \frac{1}{2} \lceil \frac{n}{5} \rceil \rfloor - 2) \geq 3(\frac{1}{2} \cdot \frac{n}{5} - 3) = \frac{3n}{10} - 9.$$

members of array a on both its left and right sides. Disect the expression to the left of $\geq$ by explaining the significance of each of the following. Three out of four correct for passing.

(a) $\frac{n}{5}$

(b) 3

(c) $\frac{1}{2}$

(d) -2

LO7. Answer the following.

(a) The Floyd-Warshall algorithm establishes a recurrence for d_{ij}^k. In words, what does d_{ij}^k equal?

(b) Provide the dynamic-programming recurrence d_{ij}^k.

(c) When executing the Floyd-Warshall algorithm, assume

$$d^4 = \begin{pmatrix} 0 & 12 & 14 & 2 & 2 & 6 \\ 9 & 0 & 20 & 13 & 1 & 3 \\ 7 & 5 & 0 & 7 & 6 & 1 \\ 15 & 10 & 19 & 0 & 5 & 2 \\ 9 & 3 & 5 & 6 & 0 & 3 \\ 6 & 5 & 4 & 8 & 2 & 0 \end{pmatrix}$$

has been computed. Use this matrix to compute d^5. Then use d^5 to compute d^6.

LO8. Answer the following.

(a) Provide the dynamic-programming recurrence for computing $d(u, v)$ the distance from vertex u to vertex v in a directed acyclic graph (DAG) $G = (V, E, c)$, where $c(e)$ gives the cost of edge e, for each $e \in E$.

(b) Draw the vertices of the following DAG G in a linear left-to-right manner so that the vertices are topologically sorted, meaning, if (u, v) is an edge of G, then u appears to the left of v. The vertices of G are a-h, while the weighted edges of G are

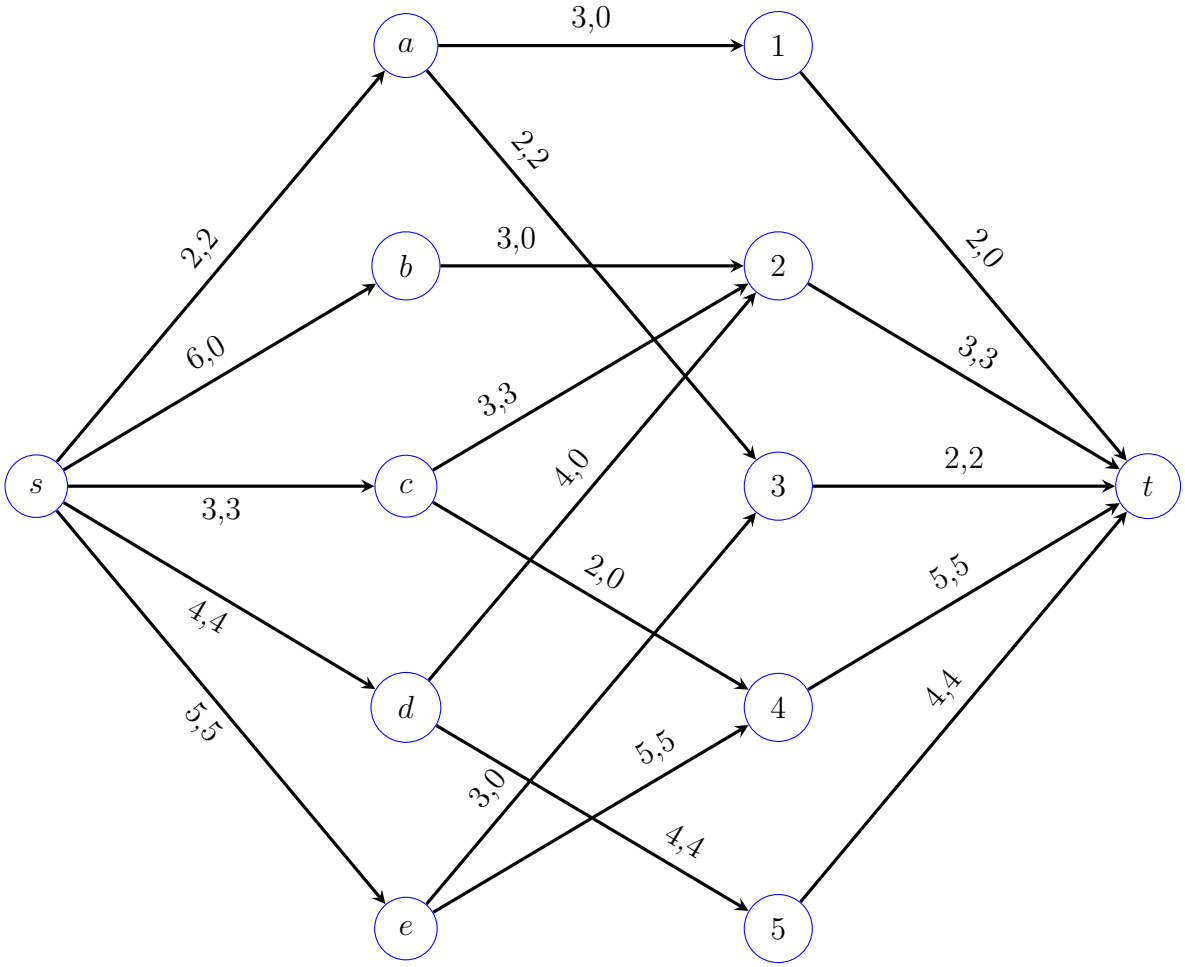
$$(a, b, 18), (a, e, 14), (a, f, 19), (b, c, 13), (b, g, 9), (c, d, 8), (c, g, 13), (c, h, 11), (d, h, 15), (e, b, 5)$$

$$(e, f, 1), (f, b, 19), (f, c, 9), (f, g, 8), (g, d, 4), (g, h, 18).$$

(c) Starting with $u = h$, and working backwards (from right to left in the topological sort), use the recurrence from part a to compute each of $d(u, h)$, where the ultimate goal is to compute $d(a, h)$.

LO9. A flow f (in red) has been placed in the network G below.

(a) Draw the residual network G_f and use it to determine an augmenting path P. Highlight path P in the network so that it is clearly visible.



(b) Redraw the original network, but with the f flow values being replaced by the $\Delta(f, P)$ flow values.

(c) What one query is needed to the Reachability-oracle in order to determine if $f_2 = \Delta(f, P)$ is a maximum flow for G?

LO10. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B.

(b) For the mapping reduction f : `Set Partition` $\rightarrow$ `Subset Sum`, determine $f(S)$ for `Set Partition` instance $S = \{4, 6, 21, 29, 30, 33, 36, 49\}$.

(c) Verify that S and $f(S)$ are both positive instances of their respective decision problems. Show work.