

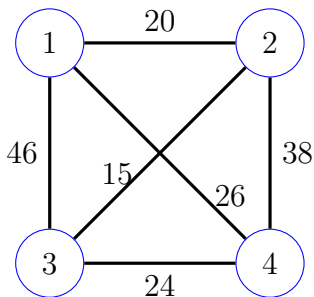
Problems

LO7. Answer/Solve the following questions/problems.

- (a) The dynamic-programming algorithm that solves the **Optimal Binary Search Tree** optimization problem defines a recurrence for the function $wac(i, j)$. In words, what does $wac(i, j)$ equal? Hint: do *not* write the recurrence (see Part b).
- (b) Provide the dynamic-programming recurrence for $wac(i, j)$.
- (c) Apply the recurrence from Part b to the keys 1-5 having respective weights 25,45,20,20,30. Show the matrix of subproblem solutions and use it to provide an optimal binary search tree.

LO8. Answer/Solve the following questions/problems.

- (a) The dynamic-programming algorithm that solves the **Runaway Traveling Salesperson** optimization problem (Exercise 30 from the Dynamic Programming Lecture) defines a recurrence for the function $mc(i, A)$. In words, what does $mc(i, A)$ equal? Hint: do *not* write the recurrence (see Part b). Note: we call it “Runaway TSP” because the salesperson does *not* return to home after visiting each city.
- (b) Provide the dynamic-programming recurrence for $mc(i, A)$.
- (c) Apply the recurrence from Part b to the graph below. Show all the necessary computations.



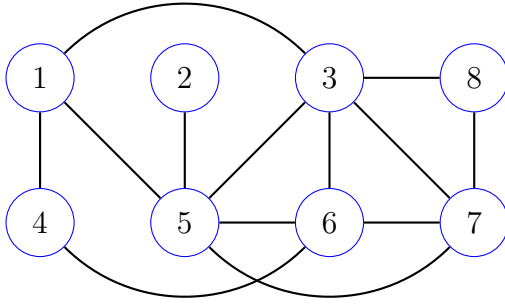
LO9. Consider the 2SAT instance

$$\mathcal{C} = \{(x_1, \bar{x}_2), (x_1, \bar{x}_3), (x_1, \bar{x}_4), (\bar{x}_1, x_3), (x_2, x_4), (\bar{x}_2, x_3), (\bar{x}_3, \bar{x}_4)\}.$$

- (a) Draw the implication graph $G_{\mathcal{C}}$.
- (b) Find a literal l for which i) R_l is an inconsistent reachability set, ii) $R_{\bar{l}}$ is a consistent reachability set, and iii) $\alpha_{R_{\bar{l}}}$ satisfies *all* the clauses of \mathcal{C} . For full credit clearly state the literal l you have chosen and verify that each of the three properties are satisfied. Hint: for example, if you choose $l = \bar{x}_3$, then $\bar{l} = \bar{\bar{x}}_3 = x_3$.

LO10. Answer the following.

- (a) Provide the definition of what it means to be a mapping reduction from decision problem A to decision problem B .
- (b) For the mapping reduction $f : \text{Clique} \rightarrow \text{Half Clique}$ provided in Exercises 11 and 12, and for **Clique** instance $(G, k = 5)$, determine $f(G, k)$, where G is shown below.



- (c) Verify that (G, k) and $f(G, k)$ are either both positive instances or both negative instances of their respective decision problems. Defend your answer.

LO11. Recall the mapping reduction $f(\mathcal{C}) = (S, t)$, where f maps an instance of **3SAT** to an instance of the **Subset** decision problem. Given **3SAT** instance

$$\mathcal{C} = \{(x_1, \bar{x}_2, x_5), (x_2, x_3, \bar{x}_4), (x_1, x_2, x_4), (\bar{x}_1, \bar{x}_3, \bar{x}_5)\}$$

answer the following questions about $f(\mathcal{C})$. Hint: to answer these questions you are *not* required to draw the table.

- (a) What is the value of t ?
- (b) How many numbers (counting repeats) are in S ? What is the largest (in terms of numerical value) number in S ?
- (c) Determine a satisfying assignment for \mathcal{C} and use it to identify a subset of S that sums to t . List all the members of S . Hint: there are multiple possible answers, but the subset you choose must correspond with your chosen satisfying assignment.