CECS 329, Homework Assignment 1, Spring 2025, Dr. Ebert

Directions: Please review the `Homework` section on page 6 of the syllabus including a list of all rules and guidelines for writing and submitting solutions.

Due Date: Tuesday February 4th as a PDF file upload to the HW1 Canvas dropbox.

Problems

1. A poster board is to have seven illustrations a through g. The caption for each illustration should be placed either directly below or above the illustration. Due to space constraints and aesthetic considerations, the poster-board's author insists on the following constraints.

   - The captions for a, b, and g should all be oriented in the same way (e.g. either all below or all above).
   - The captions for c and d should be oriented in the opposite way (i.e. one below and one above).
   - e or g (or both) should be oriented below.
   - If g is oriented below, then so is f.
   - If f is oriented below, then so is d.
   - If c is oriented below, then so is b.
   - If d is oriented below, then so are e and g.

   (a) For each of the seven constraints, provide one or more binary disjunctive clauses that are logically equivalent to the constraint. Do so using Boolean variables $\{a, b, \ldots, g\}$, where, e.g., if a is assigned 1, it means that the caption for illustration a is placed above. (10 pts)

   (b) Draw the implication graph for the set of clauses provided in part a. (5 pts)

   (c) Perform the `Improved 2SAT` algorithm to determine a way of simultaneously satisfying each of the constraints (if possible). When peforming the algorithm, compute the reachability sets in alphabetical order, first using the positive literal followed the negative literal (if necessary). Is there a way to orient the captions so that all constraints are satisfied? (10 pts)

2. Suppose $\mathcal{C}$ is a `2SAT` instance with variables x_1, x_2, x_3 and x_4. Moreover, suppose that

   $$\texttt{reachable}(G_{\mathcal{C}}, x_i, \overline{x}_i) = 0$$

   for all $i = 1, 2, 3, 4$. Is $\mathcal{C}$ satisfiable? Explain. May we conclude that

   $$\alpha = (x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 1)$$

   satisfies $\mathcal{C}$? If yes, explain your reasoning. If no, provide a *counter example*, i.e., an instance of `2SAT` that has the described property, but for which α does not satisfy $\mathcal{C}$. (15 pts)

3. Suppose you have access to a `2SAT`-oracle in that, given an instance $\mathcal{C}$ of `2SAT`, the oracle will answer 0 or 1, depending on whether or not $\mathcal{C}$ is satisfiable. Provide the description of an algorithm that takes as input a `2SAT` instance $\mathcal{C}$ and returns $\emptyset$ if $\mathcal{C}$ is unsatisfiable, or otherwise returns a satisfying assignment α for $\mathcal{C}$. To earn credit, the success of your algorithm must *entirely* depend on the `2SAT`-oracle. First describe your algorithm in a paragraph and then make it semi-formal with the help of pseudocode. (20 pts)