CECS 329, Homework Assignment 2, Spring 2025, Dr. Ebert

Directions: Please review the `Homework` section on page 6 of the syllabus including a list of all rules and guidelines for writing and submitting solutions.

Due Date: Tuesday February 11th as a PDF-file upload to the HW2 Canvas dropbox.

Problems

1. Recall that a **Hamilton Cycle (HC)** for a graph $G = (V, E)$ is a path that starts at some vertex $v \in V$, visits every other vertex of G exactly once, and then returns back to v to complete the cycle. As a traffic engineer, Frank sometimes has to compute Hamilton Cycles within transportation networks. Because of this, he has written a program that can decide if a given graph $G = (V, E)$ has an HC. His colleague Jennifer is analyzing a transportation network, modeled as a graph H, and she is interested in deciding if H has a Hamilton Path (HP) which starts at one particular location a, visits every vertex, and ends at a different location b. Frank explained to Jennifer that she could use his HC program to decide if H has an HP. "Simply connect your a and b vertices with an edge. That way, your updated graph having an HC will be equivalent to your original graph H having an HP". Give an example that shows that Frank's statement is not always true. Conclude that Frank's method is not a valid way to map reduce HP to HC. (15 pts)

2. Are you smarter than a second grader? Here is a set of numbers

$$S = \{3, 9, 14, 17, 28, 30, 34, 38, 42, 51, 74, 82, 87\}.$$

   Find a subset of them that add to $t = 250$. (10 pts)

3. Suppose you have access to a `Subset Sum` oracle, meaning that you may query the oracle about any instance (S, t) of `Subset Sum` and the oracle will return 0 or 1 depending on whether or not it is a positive instance of `SS`. Provide the description of an algorithm that takes as input a `SS` instance (S, t) and returns $\emptyset$ if S does not have a subset that sums to t, or otherwise returns such a subset $A \subseteq S$. To earn credit, the success of your algorithm must *entirely* depend on the `SS`-oracle. Describe your algorithm in a paragraph and then demonstrate it on the positive instance $(S = \{6, 12, 18, 20, 23\}, t = 47)$. Provide a table of the queries made, their answers, and how the answers lead to a solution. (25 pts)