CECS 329, Homework Assignment 3, Spring 2025, Dr. Ebert

Directions: Please review the `Homework` section on page 6 of the syllabus including a list of all rules and guidelines for writing and submitting solutions.

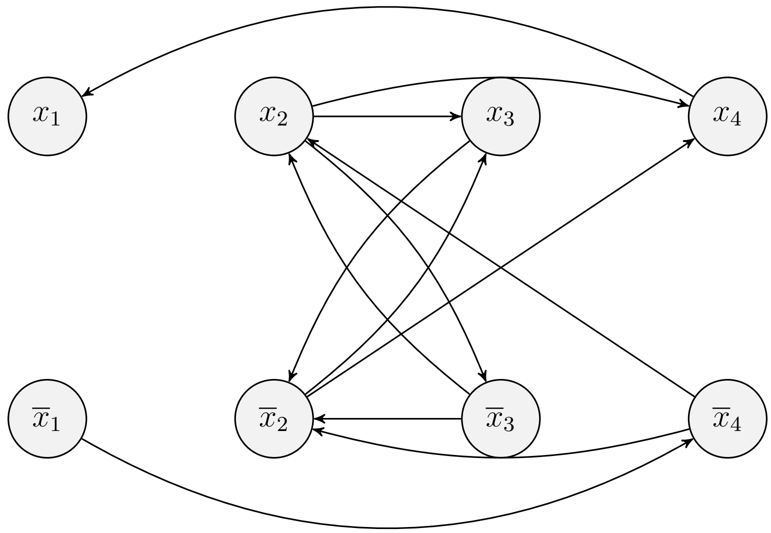
Due Date: Tuesday, February 18th as a PDF-file upload to the HW3 Canvas dropbox.

Problems

1. Suppose that a CPU of some machine M can, on average, process a single machine instruction in 5×10^{-9} seconds. Moreover, an instance of some problem is an $n \times n$ matrix R, and there are five known algorithms $(\mathcal{A}_1, \ldots, \mathcal{A}_5)$ for solving a given problem instance. When these five algorithms are programmed and compiled on M, the average number of instructions (as a function of the number n of rows and columns of matrix R) that need to be executed by each program is i) $1500n$, ii) $250n \log n$, iii) $45n^2$, iv) $6n^3$, and v) $2^{\sqrt{n}}$ respectively. What is the largest input matrix that each program can process and solve within a one-week period? Show all work for each calculation except to ii) which you may solve with the help of software. (20 points)

2. An instance of `Log Subset Sum (LSS)` is a pair (S, t), where $t > 0$ is a b-bit integer and S is a set of positive integers for which $|S| = \lfloor \log b \rfloor$. The problem is to decide if there is a subset $A \subseteq S$ whose members sum to t. In one or more paragraphs, describe an algorithm that solves the `LSS` problem in a polynomial number of steps with respect to size paramter b. In your analysis of the number of required steps, remember to include the number of steps required to add numbers. Note: writing pseudocode is unnecessary so long as your paragraph(s) clearly describe the main steps of the algorithm. (20 pts)

3. An instance of the `Feedback Arc Set (FAS)` decision problem is a pair (G, k) where $G = (V, E)$ is a directed graph and k is a nonnegative integer. The problem is to decide if there is a subset E' of k edges that can be removed from G so that $G - E'$ is an acyclic graph. As an example consider the graph $G_{\mathcal{C}}$ shown below taken from the `2SAT` lecture. This graph has two cycles of length two: $C_1 = x_3, \overline{x}_2, x_3$ and $C_2 = x_2, \overline{x}_3, x_2$, but if we remove edges $e_1 = (x_3, \overline{x}_2)$ and $e_2 = (x_2, \overline{x}_3)$ from $G_{\mathcal{C}}$, then it becomes an acyclic graph. Therefore $(G_{\mathcal{C}}, k = 2)$ is a positive instance of `FAS`.



(a) For a given instance (G, k) of `FAS`, describe a certificate in relation to (G, k). (5 pts)

(b) Provide a semi-formal verifier algorithm that takes as input i) an instance (G, k), ii) a certificate for (G, k) as defined in part a, and decides if the certificate is valid for (G, k). Do this by making use of a reachability oracle via function `reachable`(H, a, b) which returns true iff vertex b is reachable from vertex a in graph H. Hint: for each vertex $v \in V$, consider the set $N^+(v) = \{u|(u, v) \in E\}$. (10 pts)

(c) Provide size parameters that may be used to measure the size of an instance (G, k) of `FAS`. (5 pts)

(d) Use the size parameters from part c to describe the running time of your verifier from part b. Assume that a single call to function `reachable` requires a linear number of steps with respect to the size of the input graph. Defend your answer in relation to the algorithm you provided for the verifier. (5 pts)