CECS 528, Homework Assignment 3, Spring 2025, Dr. Ebert

Directions: Please review the `Homework` section on page 6 of the syllabus including a list of all rules and guidelines for writing and submitting solutions.

Due Date: Monday, February 17th as a PDF-file upload to the HW3 Canvas dropbox.

Problems

1. Given an array $a[0 : n-1]$ of n positive integers, the goal is to determine the subarray $a[i : j]$ for which
$$Q(i,j) = (j - i + 1) \min_{i \le k \le j} (a[k])$$
is maximized, where $0 \le i \le j \le n-1$. For example if $a = 3, 3, 1, 7, 4, 2, 4, 6, 1$, then $a[3 : 7]$ yields the maximum value, since the minimum a-value in this range is $a[5] = 2$, and
$$Q(3,7) = (7 - 3 + 1)(2) = 10.$$
One algorithm for finding $a[i : j]$ is to consider all n^2 possible combinations of i and j and keep track of the combination that produces the maximum value. But this algorithm has quadratic running time (why?). Instead, provide a greedy algorithm that can determine the optimal subarray $a[i : j]$ in $O(n \log n)$ steps.

   (a) Clearly describe your algorithm in a few paragraphs, followed by pseudocode. Make sure all variables are clearly defined. (15 pts)

   (b) Prove that your algorithm's running time $O(n \log n)$. (5 pts)

   (c) Demonstrate how your algorithm works on the array $a = 5, 9, 10, 2, 8, 3, 7, 1, 6, 4$. Do this by providing a table whose rows represent each stage of the algorithm and whose columns keep track of all necessary variables and structures that are needed to illustrate the algorithm. (10 pts)

   (d) Using one or more paragraphs, prove that your algorithm will always find the optimal subarray. Note: you may or may not find the Replacement method helpful. It depends on your algorithm. (10 pts)

2. A third algorithm for finding an MST in a weighted graph $G = (V, E, c)$ works as follows. Sort the edges of E from highest to lowest weight. For each edge e taken in sorted order, if removing e does *not* disconnect G, then remove e from E:
$$E \leftarrow E - \{e\}.$$
Otherwise, e remains in E. Then after this procedure, what remains in E is an mst for the original graph G. Use the Replacement method to prove the correctness of this algorithm. Hint: for guidance, study the correctness proofs for Kruskal's and Prim's algorithm. ((15 points)