

CECS 528, Homework Assignment 5, Fall 2024, Dr. Ebert

Directions: Please review the Homework section on page 6 of the syllabus including a list of all rules and guidelines for writing and submitting solutions.

Due Date: Friday March 21st as a PDF-file upload to the HW5 Canvas dropbox.

Problems

1. Consider the following decision problem which we'll call **I**. An instance of **I** is i) a collection T of distinct nonempty subsets (i.e. no two subsets are the same) of $\{1, \dots, n\}$, for some $n \geq 1$ and ii) a nonnegative integer $k \geq 0$. The problem is to decide if there are k members A_1, \dots, A_k of T for which $A_i \cap A_j = \emptyset$, for all $1 \leq i < j \leq k$. In other words A_1, \dots, A_k are pairwise disjoint subsets.

(a) Show that (T, k) is a positive instance of **I**, where

$$T = \{\{10, 11\}, \{12, 14, 16\}, \{10, 11, 12\}, \{2, 3, 5, 9, 10, 19\}, \{2, 3, 5, 6\}, \{4, 5, 6\},$$

$$\{18, 19\}, \{6, 7\}, \{9, 10, 15, 19\}, \{2, 3, 4, 6, 11\}, \{6, 7, 10, 17, 18\}, \{3, 4, 5\}\}$$

and $k = 4$. Provide A_1, \dots, A_4 . (5 pts)

- (b) An instance of decision problem **Half-I** is a collection T of distinct subsets of $\{1, \dots, n\}$, for some $n \geq 1$. The problem is to decide if at least one half of the subsets of T are pairwise disjoint. Provide an efficient mapping reduction $f : \mathbf{I} \rightarrow \mathbf{Half-I}$ from **I** to **Half-I**. Prove that your mapping reduction is correct by arguing that, for any possible (T, k) , the answer to (T, k) is always the same as the answer for $T' = f(T, k)$. (20 pts)
2. An instance of **Log Subset Sum (LSS)** is a pair (S, t) , where $t > 0$ is a b -bit integer and S is a set of positive integers for which $|S| = \lfloor \log b \rfloor$. The problem is to decide if there is a subset $A \subseteq S$ whose members sum to t . In one or more paragraphs, describe an algorithm that solves the **LSS** problem in a polynomial number of steps with respect to size parameter b . In your analysis of the number of required steps, remember to include the number of steps required to add numbers. Note: writing pseudocode is unnecessary so long as your paragraph(s) clearly describe the main steps of the algorithm. (20 pts)