CECS 528, LO Assessment 1, 1/29/25, Dr. Ebert

Directions: show all work.

Problems

LO1. Complete the following problems.

$\log_5 32$ $\boxed{0 < \varepsilon < \log_5 32 - 2}$

(a) Use the Master Theorem to determine the growth of $T(n)$ if it satisfies the recurrence $T(n) = 32T(n/5) + n^2 \log^3 n$.

$n^{\log_5 32} = n^{2+\delta}$, for some $\delta > 0$. $\underline{n^2 \log^3 n} =$ $\underline{O(n^{2+(\delta-\varepsilon)})}$ for some $0<\varepsilon<\delta$. $\therefore$ By Case 1 of M.T. $T(n) = \Theta(n^{\log_5 32})$

(b) Use the substitution method to prove that, if $T(n)$ satisfies

$$T(n) = 4T(n/2) + 7n^2,$$

Then $T(n) = \Omega(n^2 \log n)$.

Assume $T(k) \geq \underline{ck^2 \log k}$ for some const. $c > 0$ and all $k < n$.

Show: $\underline{T(n) \geq cn^2 \log n}$

$\underline{T(n)} = 4T(\underbrace{n/2}_{k}) + 7n^2 \geq 4c\left(\frac{n}{2}\right)^2 \log_2\left(\frac{n}{2}\right) + 7n^2$

$= cn^2(\log n - 1) + 7n^2 = \boxed{cn^2 \log n} - cn^2 + 7n^2$

$\geq \boxed{cn^2 \log n} \iff cn^2 \leq 7n^2 \iff c \leq 7$