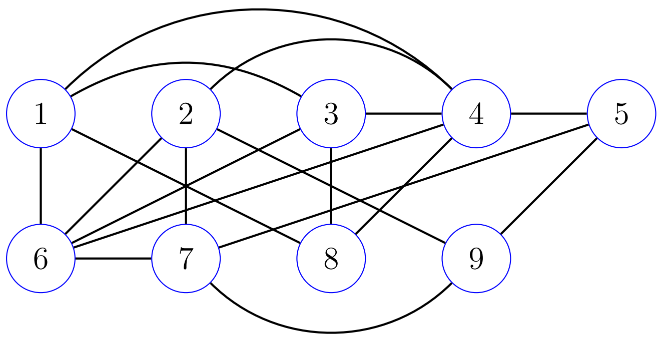
CECS 528, Learning Outcome Assessment 9, 4/16/2025, Dr. Ebert

Problem

LO6. Do the following.

(a) Provide the definition of what it means to be a mapping reduction from problem A to problem B.

(b) The simple graph $G = (V, E)$ shown below along with $k = 4$ is an instance of the `Clique` decision problem. Draw $f(G, k)$, where f is the mapping reduction from `Clique` to `Half Clique` provided in one of the lecture exercises.



(c) Verify that f is valid for input (G, k) from part b in the sense that both (G, k) and $f(G, k)$ are either both positive instances or both negative instances. Make sure your answer is specific to the particular instances from part b.

LO7. An instance of the `Max Cut` decision problem is a weighted simple graph $G = (V, E, w)$ and an integer $k \geq 0$. The problem is to decide if there is a subset $U \subseteq V$ for which

$$w(e_1) + w(e_2) + \cdots + w(e_r) \geq k,$$

where $\{e_1, e_2, \ldots, e_r\}$ is the set of all edges e for which one vertex of e is a member of U, and the other vertex is a member of $E - U$.

(a) For a given instance (G, k) of `Max Cut`, describe a certificate in relation to (G, k).

(b) Provide a semi-formal verifier algorithm that takes as input i) an instance (G, k) of `Max Cut`, ii) a certificate for (G, k) as defined in part a, and decides if the certificate is valid for (G, k).

(c) Assuming that $w : E \to \mathcal{N}$ maps edges to natural numbers, clearly define three size parameters for the `Max Cut` decision problem.

(d) Use the size paramters from part c to describe the running time of your verifier. Defend your answer.

LO8. Answer the following.

(a) Consider the mapping reduction $f : \texttt{3SAT} \to \texttt{DHP}$ from 3SAT to DHP that was presented in lecture and the 3SAT instance

$$\mathcal{C} = \{c_1 = (x_1, x_2, x_3), c_2 = (\overline{x}_2, x_3, \overline{x}_4), c_3 = (x_1, x_2, \overline{x}_4), c_4 = (\overline{x}_1, \overline{x}_3, \overline{x}_4), c_5 = (\overline{x}_1, x_2, x_4)\}.$$

i. Draw the subgraph of $f(\mathcal{C})$ whose vertices are lc_1, rc_1, and c_1, where lc_1 and rc_1 belong to the chain of vertices in the x_1-diamond and c_1 is the clause-1 vertex. Explain your drawing.

ii. Which diamond of $f(\mathcal{C})$ does not have edges connecting to clause vertex c_4? Explain.

iii. Is there a Hamilton path from a to b in $f(\mathcal{C})$ that moves from left to right through each of the diamonds of $f(\mathcal{C})$? Explain

(b) Consider the mapping reduction $f : \texttt{3SAT} \to \texttt{Clique}$ from 3SAT to Clique that was presented in lecture. Let $\mathcal{C}$ be the 3SAT instance from part a) of this LO. How many vertices and edges does the graph of $f(\mathcal{C})$ have? Show work.

LO9. Do the following.

(a) Draw the set

$$S = \{(0,0), (0,9)(1,6), (2,5), (2,7), (3,2), (3,4), (3,6), (3,9), (4,1), (5,1),$$
$$(5,2), (5,8), (7,3), (7,7), (8,4), (9,8), (10,0), (10,7)\}$$

of points and apply the `k-Clustering` algorithm to S and $k = 4$. Select $(0, 9)$ as the first center. Clearly indicate which points are cluster centers and enclose each cluster with a boundary curve. Label each cluster (e.g. C_1) in accordance with the order in which it's center was selected. Note: all distances are Euclidean: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

(b) Answer the following regarding the anaysis of Christofides' approximation algorithm for TSP, assuming that edge distances satisfy the triangle inequality.

i. Christofides' algorithm requires performing an Euler tour through some graph. What are the two sources of edges for this graph?

ii. Given an optimal TSP Hamilton cycle $C = b, f, d, c, e, i, a, h, l, g, b$, provide a corresponding Hamilton cycle C' for the complete subgraph whose vertices are $\{e, f, g, h\}$ and for which $\mathrm{cost}(C') \le \mathrm{cost}(C)$. Why is this inequality true for your provided cycle?

iii. Christofides' algorithm provides an approximation ratio of 1.5. Use part ii) to explain how the 0.5 is obtained.

LO10. An urn consists of six balls: 2 reds, 2 blues, and 2 greens. Suppose two balls are randomly selected from the urn. Let R, B, and G denote the number of reds, blues, and greens that were selected, respectively.

(a) Provide the domain and probability distribution for each of these random variables. Show work.

(b) Compute both $E[R]$ and $E[R|B = 1]$. Note: all answers should be in the form of fractions.

(c) Suppose after selecting the balls we keep them if they are of the same color, or place them back in the urn and randomly re-select. This process continues until we have selected two balls of the same color. How many times should we expect to have to select from the urn before selecting two balls of the same color? Show work and explain.