CECS 528, Learning Outcome Assessment 2, 2/5/2025, Dr. Ebert

Directions: show all work.

Problem

LO1. Solve the following problems.

(a) Use the Master Theorem to determine the growth of $T(n)$ if it satisfies the recurrence $T(n) = 8T(n/2) + n^3 \log^2 n$. Defend your answer.

$n^{\log_2 8} = n^3$ $\quad f(n) = n^3 \log^2 n.$

$\therefore$ By Case 4 of M.T.,

$T(n) = \Theta(n^3 \log^3 n)$

(b) Use the substitution method to prove that, if $T(n)$ satisfies

$$T(n) = T(n/5) + T(3n/10) + n$$

then $T(n) = \mathrm{O}(n)$.

Inductive Assumption:
$T(n) \leq ck$, for all $k < n$ and some const. $c > 0$.

Show $T(n) \leq cn$

$T(n) = T(n/5) + T(3n/10) + n \leq c\left(\frac{n}{5}\right) + c\left(\frac{3n}{10}\right) + n$

$= \frac{2cn}{10} + \frac{3cn}{10} + n = \frac{cn}{2} + n \leq cn \Leftrightarrow$

$c \geq 2$

LO2. Solve each of the following problems.

(a) Recall that the `Median-of-Five Find-Statistic` algorithm makes use of the Partitioning algorithm and uses a pivot that is guaranteed to have at least

$$3(\lfloor \frac{1}{2} \lceil \frac{n}{5} \rceil \rfloor - 2) \geq 3(\frac{1}{2} \cdot \frac{n}{5} - 3) = \frac{3n}{10} - 9$$

members of array a both to its left and to its right. Rewrite each of the above inequalities but now assuming the algorithm uses groups of eleven instead of groups of five. **Provide a justification for each of the numerical changes.**

5 → 11 : Groups of 11 instead of 5

3 → 6 : For example, If the median of some group of 11 is ≤ pivot, then so are 5 other members of that group.

$5+1=\underline{6}$

$$6(\lfloor \frac{1}{2} \lceil \frac{n}{11} \rceil \rfloor - 2) \geq 6\left(\frac{n}{22} - 3\right) = \boxed{\frac{3n}{11} - 18}$$

(b) Let (a, k) be an instance of the `Median-of-Five Find-Statistic` algorithm, where

$$a = \boxed{20, 57, 73, 83, 32}, \boxed{78, 82, 47, 12, 97}, \boxed{23, 76, 52, 40, 58}, \boxed{10, 91, 51, 86, 37}, \boxed{76, 73, 74},$$

$M_1 = 57$ $M_2 = 78$ $M_3 = 52$ $M_4 = 51$ $M_5 = 74$

and $k = 12$. Determine the pivot for the partitioning step when performed at the top level of recursion. After the partitioning step is completed, has the $k = 12$ statistic been found? Explain. If not provide the lower and upper indices of a for where the $k = 12$ statistic must now lie.

Pivot M = median(57, 78, 52, 51, 74) = 57

After Partitioning Step: $|a_{left}| = 10$

$|a_{right}| = 12$. Thus M is located at index $\underline{10}$

$12 > 10 \Rightarrow$ a_{right} is next array with index range

lower = 11 upper = 22