CECS 528, Learning Outcome Assessment 2, 2/5/2025, Dr. Ebert

Directions: show all work.

Problem

LO1. Solve the following problems.

(a) Use the Master Theorem to determine the growth of $T(n)$ if it satisfies the recurrence $T(n) = 8T(n/2) + n^3 \log^2 n$. Defend your answer.

(b) Use the substitution method to prove that, if $T(n)$ satisfies

$$T(n) = T(n/5) + T(3n/10) + n$$

then $T(n) = \mathrm{O}(n)$.

LO2. Solve each of the following problems.

(a) Recall that the `Median-of-Five Find-Statistic` algorithm makes use of the Partitioning algorithm and uses a pivot that is guaranteed to have at least

$$3(\lfloor\frac{1}{2}\lceil\frac{n}{5}\rceil\rfloor - 2) \geq 3(\frac{1}{2}\cdot\frac{n}{5} - 3) = \frac{3n}{10} - 9$$

members of array a both to its left and to its right. Rewrite each of the above inequalities but now assuming the algorithm uses groups of eleven instead of groups of five. **Provide a justification for each of the numerical changes.**

(b) Let (a, k) be an instance of the `Median-of-Five Find-Statistic` algorithm, where

$$a = 20, 57, 73, 83, 32, 78, 82, 47, 12, 97, 23, 76, 52, 40, 58, 10, 91, 51, 86, 37, 76, 73, 74,$$

and $k = 12$. Determine the pivot for the partitioning step when performed at the top level of recursion. After the partitioning step is completed, has the $k = 12$ statistic been found? Explain. If not provide the lower and upper indices of a for where the $k = 12$ statistic must now lie.