CECS 528, Learning Outcome Assessment 3, 9/18/2024, Dr. Ebert

Directions: show all work.

Problem

LO1. Solve the following problems.

(a) Use the Master Theorem to determine the growth of $T(n)$ if it satisfies the recurrence $T(n) = 8T(n/2) + n^3$. Defend your answer.

$T(n) = \Theta(n^3 \log n)$ by Case 2 of M.T.

(b) Use the substitution method to prove that, if $T(n)$ satisfies

$$T(n) = 2T(n/2) + n \log n$$

then $T(n) = \Omega(n \log^2 n)$.

Inductive Assumption: $T(k) \geq ck\log^2 k$ for all $k<n$ and some const. $c>0$. Show $T(n) \geq cn\log^2 n$.

$T(n) = 2T(n/2) + n\log n \geq 2c\left(\frac{n}{2}\right)\log^2\left(\frac{n}{2}\right) + n\log n =$
$cn(\log n - 1)^2 + n\log n = cn\log^2 n - 2cn\log n + cn$
$+ n\log n \geq cn\log^2 n \iff$

$c(2n\log n - n) \leq n\log n \iff$

$c \leq \dfrac{n\log n}{2n\log n - n} = \dfrac{1}{2 - \frac{1}{\log n}}$ which is

true for all $n \geq 1$ iff $c \leq \frac{1}{2}$

LO2. Solve each of the following problems.

(a) Recall that the `Find-Statistic` algorithm makes use of the Partitioning algorithm and uses a pivot that is guaranteed to have at least

$$3(\lfloor\frac{1}{2}\lceil\frac{n}{5}\rceil\rfloor - 2) \geq 3(\frac{1}{2}\cdot\frac{n}{5} - 3) = \frac{3n}{10} - 9 \geq \frac{n}{4}$$

members of array a both to its left and to its right. Rewrite each of the above inequalities but now assuming the algorithm uses groups of nine instead of groups of five. **Explain your reasoning for each of the numerical changes that you make**. How large does n have to be in order for the 2nd rewritten inequality to be true.

$\frac{n}{5} \Rightarrow \frac{n}{9}$ since there are now $\lceil\frac{n}{9}\rceil$ groups.

$3 \Rightarrow 5$ since for every group median that is $\leq$ ($\geq$) the pivot, there are four other group members who are $\leq$ ($\geq$) the pivot as well. Also, $\frac{5n}{18} - 15 \geq \frac{n}{4} \Leftrightarrow$ $10n - (36)(15) \geq 9n \Leftrightarrow$ $n \geq (36)(15) = 540$

(b) Demonstrate the partitioning step of Hoare's Quicksort for the array

$$a = 16, 2, 1, 5, 6, 3, 13, 10, 7, 12, 18, 10, 19.$$

Use the median-of-three method to select the pivot. M = median(16, 13, 19) = 16

10
~~19~~

16 19 ~~16~~ 18

a = ~~16~~, 2, 1, 5, 6, 3, 13, 10, 7, 12, ~~18~~, ~~10~~, ~~19~~

$\uparrow l_1$ $\uparrow r_2$ l_2 $\uparrow r_1$

a_{Left} = 10, 2, 1, 5, 6, 3, 13, 10, 17, 12

a_{Right} = 19, 18

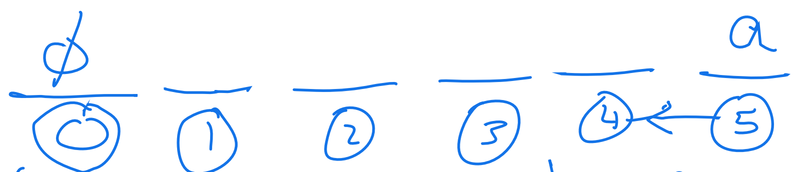
LO3. Solve each of the following problems.

(a) Recall the use of the disjoint-set data structure for the purpose of improving the running time of the `Unit Task Scheduling` algorithm. For the set of tasks

Task	a	b	c	d	e	f
Deadline Index	5	5	4	4	5	4
Profit	60	50	40	30	20	10

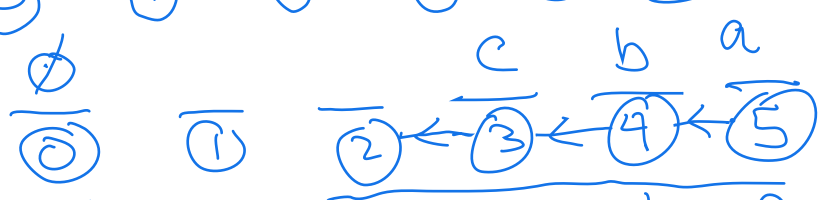
For each task, show the M-Tree forest after it has been inserted (or at least has attempted to be inserted in case the scheduling array is full). Note that the earliest possible deadline index is 1, meaning that the earliest slot in the schedule array has index 1. Also, assume that an insert attempt that takes place at index i results in the function call `root`(i). Finally, to receive credit, your solution should show six different snapshots of the M-Tree forest.

Insert a:

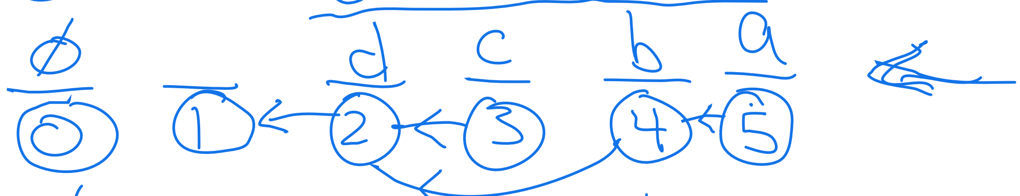


Insert b:

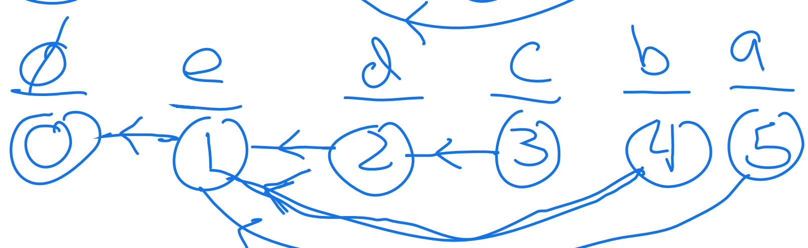
Insert c:



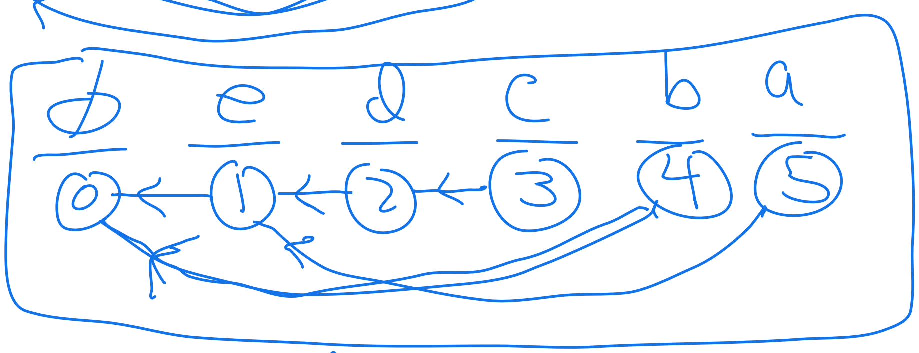
Insert d:



Insert e:



Insert f (fail):



final state

(b) Demonstrate Kruskal's algorithm on the set of edges

$\{(1,2,38), (1,6,29), (1,7,22), (1,8,25), (2,3,37), (2,9,86), (2,6,53), (3,5,48), (3,7,48), (3,8,52), (3,10,64), (4,5,18), (4,7,53), (5,6,42), (6,10,88), (7,9,83), (8,9,73), (9,10,54)\}$.

Assume that the algorithm terminates once a spanning tree has been formed. Circle all edges in the set that the algorithm attempted to add to the forest, but were rejected.