CECS 528, Learning Outcome Assessment 6, 10/16/2024, Dr. Ebert

Problem

LO2. Solve each of the following problems.

(a) When performing the FFT algorithm for the purpose of evaluating the polynomial

$$p(x) = -8 + 5x + 4x^2 + 6x^3 - 2x^4 + 9x^5 + 3x^6 - 5x^7,$$

at the 8th roots of unity, the algorithm recursively calls FFT on two subproblem instances. What is the polynomial for each instance. In relation to these recursive calls, why is it important that the squares of the 8th roots of unity yield the 4th roots of unity?

(b) Let (a, k) be an instance of the `Median-of-Five Find-Statistic` algorithm, where

$$a = 16, 13, 75, 76, 1, 12, 53, 13, 2, 67, 60, 35, 43, 12, 58, 35, 19, 75, 18, 62, 38, 72, 55,$$

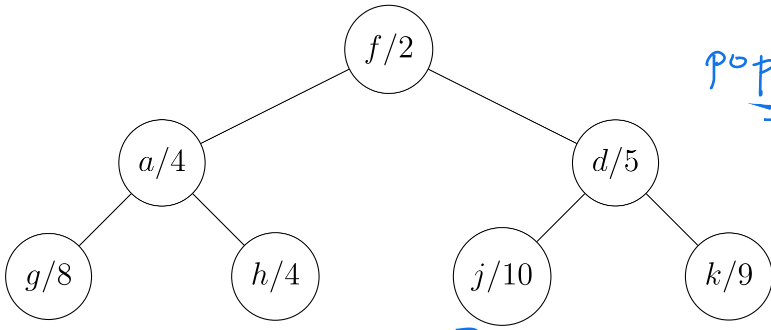
and $k = 16$. Determine the pivot for the partitioning step when performed at the top level of recursion. After the partitioning step is completed, has the $k = 16$ statistic been found? Explain. If not provide the lower and upper indices of a for where the $k = 16$ statistic must now lie.

LO3. Solve each of the following problems.

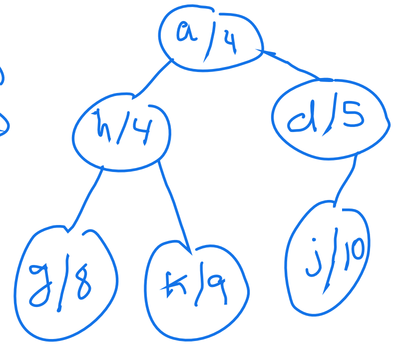
(a) The tree below shows the state of the binary min-heap at the beginning of some round of Dijkstra's algorithm, applied to some weighted graph G. If G has directed edges

$$(a, f, 1), (d, g, 2), (f, g, 5), (f, h, 2), (f, k, 3),$$

then draw a plausible state of the heap at the end of the round.

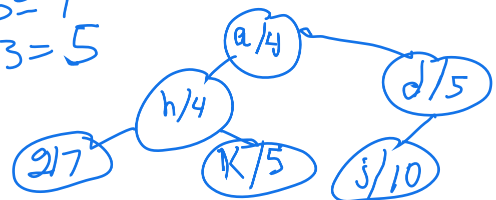


pop() $\Longrightarrow$



Increase priority: $g: 8 \to 2+5=7$
$k: 9 \to 2+3=5$

final heap $\Longrightarrow$



(b) Demonstrate the `Fractional Knapsack` algorithm using the input items $1-6$ whose respective weights are $3, 3, 4, 2, 5, 2$ and respective profits are $20, 50, 30, 10, 20, 50$. Assume a knapsack capacity equal to $M = 10$. Show all work.

profit per unit weight ratios: $6\frac{2}{3}, 16\frac{2}{3}, 7\frac{1}{2}, 5, 4, 25$
1 2 3 4 5 6

Items sorted by profit-to-weight ratios: 6, 2, 3, 1, 4, 5

Final Knapsack load: Items 6, 2, 3, 1 (1/3 of unit 1)

Total Profit $= 50 + 50 + 30 + 6\frac{2}{3} =$ $136\frac{2}{3}$

LO4. Solve the following problems.

(a) The dynamic-programming algorithm that solves the `Longest Common Subsequence (LCS)` optimization problem defines a recurrence for the function lcs(i, j). In words, what does lcs(i, j) equal? Hint: do *not* write the recurrence (see Part b). See Notes

(b) Provide the dynamic-programming recurrence for lcs(i, j).

(c) Apply the recurrence from Part b to the words u = baaabb and v = aaabbb. Show the matrix of subproblem solutions and use it to provide an optimal solution.

$$lcs(i,j) = \begin{cases} 0 & \text{if } i=0 \text{ or } j=0 \\ \max(lcs(i-1,j), lcs(i,j-1)) & \text{if } u_i \neq v_j \\ lcs(i-1,j-1)+1 & \text{otherwise} \end{cases}$$

lcs

$i \setminus j$	λ_0	a_1	a_2	a_3	b_4	b_5	b_6
λ_0	0	0	0	0	0	0	0
b_1	0	0	0	0	↖ 1	↖ 1	↖ 1
a_2	0	↖ 1	↖ 1	↖ 1	← 1 ↑	← 1 ↑	← 1 ↑
a_3	0	↖ 1	↖ 2	↖ 2	← 2	← 2	← 2
a_4	0	↖ 1	↖ 2	↖ 3	← 3	← 3	← 3
b_5	0	↑ 1	↑ 2	↑ 3	↖ 4	↖ 4	↖ 4
b_6	0	1 ↑	↑ 2	3 ↑	↖ 4	↖ 5	↖ 5

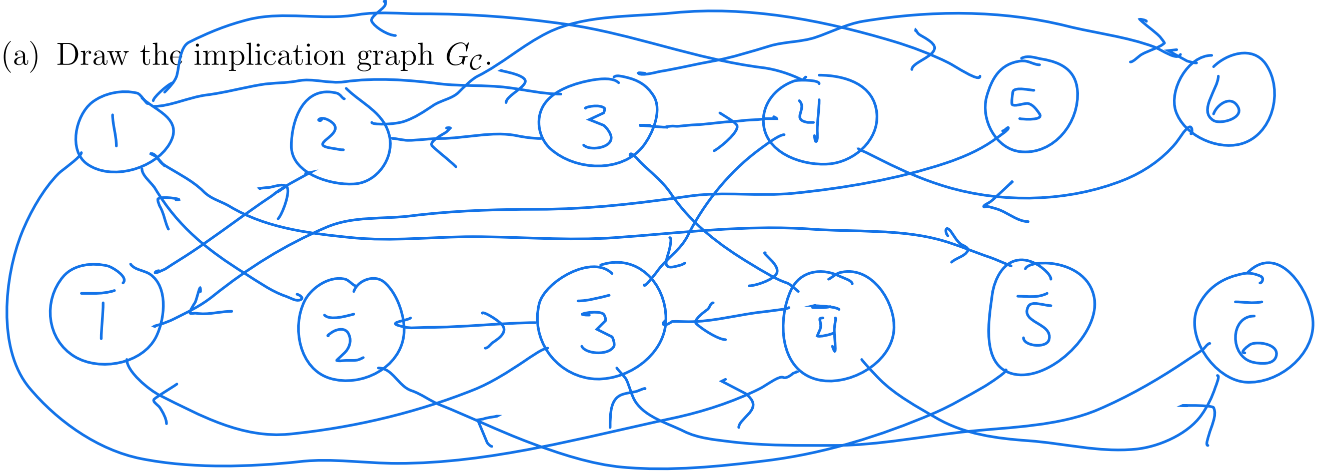
baaabb

aaabbb

LO5. Consider the 2SAT instance

$$\mathcal{C} = \{(x_1, x_2), (\overline{x}_1, x_3), (\overline{x}_1, \overline{x}_4), (\overline{x}_1, \overline{x}_5), (x_2, \overline{x}_3), (\overline{x}_2, x_5), (\overline{x}_3, x_4), (\overline{x}_3, \overline{x}_4), (\overline{x}_3, x_6), (x_4, \overline{x}_6)\}.$$

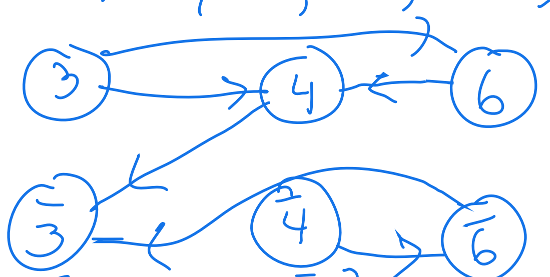
(a) Draw the implication graph $G_{\mathcal{C}}$.

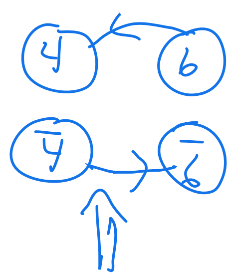


(b) Perform the `Improved 2SAT` algorithm by computing the necessary reachability sets. Use numerical order (in terms of the variable index) and positive literal before negative literal when choosing the reachability set to compute next. Draw the resulting reduced `2SAT` instance whenever a consistent reachability set is computed. Either provide a final satisfying assignment for $\mathcal{C}$ or indicate why $\mathcal{C}$ is unsatisfiable.

$R_{x_1} = \{x_1, x_3, x_2, x_5, \overline{x}_1, x_4, \overline{x}_3, x_6, \overline{x}_5, \overline{x}_2, \overline{x}_4, \overline{x}_6\}$ inconsistent

$R_{\overline{x}_1} = \{\overline{x}_1, x_2, x_5\}$ is consistent. $\alpha_{R_{x_1}} = (x_1 = 0, x_2 = 1, x_5 = 1)$





$R_{x_4} = \{x_4\}$ $\Downarrow$ $\alpha_{R_{x_4}} = (x_4 = 1)$

6 $\alpha_{R_{x_6}} = (x_6 = 1)$

$R_{x_3} = \{x_3, x_4, x_6, \overline{x}_3\}$ is inconsistent. $R_{\overline{x}_3} = \{\overline{x}_3\} \Rightarrow \alpha_{R_{\overline{x}_3}} = (x_3 = 0)$

(c) instance $\mathcal{C}$ of `2SAT` has 400 variables and 2,498 different clauses. For the original `2SAT` algorithm, what is the *least* possible number of `Reachability`-oracle queries that are required before one can conclude with certainty that $\mathcal{C}$ is satisfiable? Explain.

Final Assignment: $\alpha = (x_1 = 0, x_2 = 1, x_3 = 0, x_4 = 1, x_5 = 1, x_6 = 1)$

400 in case $\text{Reachable}(G_{\mathcal{C}}, x_i, \overline{x}_i) = 0$ for all $i = 1, \ldots, 400$

LO6. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from problem A to problem B.

See Notes

(b) Given `Set Partition (SP)` instance $S = \{4, 6, 7, 16, 21, 25, 30, 37, 42\}$, provide $f(S)$, where $f : \texttt{SP} \to \texttt{SS}$ is the mapping reduction from `SP` to `Subset Sum` described in lecture.

$f(S) = (S, t = \frac{M}{2}) = (S, 94)$ where $M = \sum_{s \in S} s = 188$.

(c) Verify that f is valid for input S from part b in the sense that both S and $f(S)$ are positive instances of their respective problems. Make sure to provide the solutions for both problem instances.

S is positive for SP since

$A = \{4, 7, 21, 25, 37\}$, $B = \{6, 16, 30, 42\}$

is a partition of S with $\sum_{a \in A} = \sum_{b \in B} = 94$

$f(S) =$

Also, $(S, t = 94)$ is positive for SS via subset A.