

Final Exam Review

Last Updated: May 8th, 2025

Algorithm Fundamentals

Definitions. computational problem, problem instance, size parameters, big-O notation (O , Ω , Θ , o , ω), recurrence relation, divide-and-conquer recurrence (uniform versus non-uniform), recursion trees (sub-problem trees versus complexity trees), optimal substructure, memoization.

Algorithms. Questions to ask: what is the computational problem that is solved by this algorithm? What is the “big-picture” idea of how the algorithm works? What is its worst-case running time? What conditions cause the worst-case to occur? How to prove that the algorithm always returns a correct answer?

Fundamental Algorithmic Paradigms. Divide and Conquer, Greedy, Dynamic Programming, Mapping Reductions

Divide and Conquer Algorithms. Mergesort, Quicksort, Find-Statistic, Strassen’s algorithm, Karatsuba’s Integer Multiplication algorithm, Maximum Subsequence Sum, and Minimum Positive Subsequence Sum, Fast Fourier Transform. Questions to ask: what is the divide-and-conquer recurrence associated with the algorithm?

Greedy Algorithms. Kruskal, Prim, and Dijkstra’s algorithms, Unit Task Scheduling (UTS), Task Selection, Fractional Knapsack, Fuel Reloading. Questions to ask: how can a data structure be used to improve the algorithm’s running time?

Data Structures for Greedy Algorithms. Disjoining Set data structure (Kruskal and UTS), Binary Heap (Prim and Kruskal).

Dynamic Programming Algorithms. 0-1 Knapsack, Edit Distance, Longest Common Subsequence, Optimal Binary Search Tree, Single-source Shortest and Longest Paths in an Acyclic Graph, Floyd-Warshall, Traveling Salesperson. Questions to ask: what is the dynamic-programming recurrence associated with the algorithm.

Substitution Method. inductive assumption, substitution procedure

Theorems. Master Theorem

Complexity Theory

Definitions. Turing reducibility, polynomial-time Turing reducibility, oracle, oracle query, mapping reducibility, polynomial-time mapping reducibility, problem domain, computational complexity theory, decision problem, optimization problem, problem instance, positive/negative instances, size parameter, P, NP, NP-completeness, P =? NP open problem, predicate function

Turing Reductions. Multiply \leq_T Add, 2SAT \leq_T^p Reachability, Max Flow \leq_T^p Reachability

Logic Problems and Related Definitions. SAT, 2SAT, 3SAT, Boolean formula, clause, implication, contrapositive, variable, literal, consistent/inconsistent literal set, variable assignment, satisfiable, unsatisfiable

Mathematical Problem Domains. Sets, Logic, Graph Theory, Number Theory

Three Kinds of Mapping Reductions. Embeddings, Contractions, Interdomain

Examples of Mapping Reductions.

- Even \leq_m^p Odd
- Independent Set \leq_m^p Clique
- Clique \leq_m^p Independent Set
- Independent Set \leq_m^p Vertex Cover
- Set Partition \leq_m^p Subset Sum
- Hamilton Path \leq_m^p LPath
- Subset Sum \leq_m^p Set Partition
- Vertex Cover \leq_m^p Half Vertex Cover
- Clique \leq_m^p Half Clique
- SAT \leq_m^p 3SAT
- 3SAT \leq_m^p Clique
- 3SAT \leq_m^p Subset Sum

- $3SAT \leq_m^p \text{Directed Hamilton Path}$
- $\text{Directed Hamilton Path} \leq_m^p \text{Hamilton Path}$
- $\text{Hamilton Path} \leq_m^p \text{Hamilton Cycle}$
- $\text{Hamilton Cycle} \leq_m^p \text{Traveling Salesperson}$
- $\text{Max Matching} \leq_m^p \text{Max Flow}$

Notable Problems in P. Reachability, 2SAT, Prime, Distances in Graphs, Boolean Formula Evaluation, Max Flow, Max Matching

How to Prove that a problem is in NP? certificate set, certificate, valid certificate

How to Prove that an NP problem is NP-complete? NP-completeness reductions

Notable NP-complete problems. SAT, 3SAT, Clique, Half Clique, Independent Set, Subset Sum, 3-Dimensional Matching, Set Partition, Vertex Cover, Half Vertex Cover, Directed Hamilton Path, Hamilton Path, Hamilton Cycle, Traveling Salesperson

Theorems. Cook's Theorem, Transitivity of both Turing and Mapping reducibility

Practice Problems

- Which divide-and-conquer recurrence best describes the running time of **Strassen's** algorithm.
 - $T(n) = 4T(n/2) + n^3$
 - $T(n) = 8T(n/2) + n^2$
 - $T(n) = 7T(n/2) + n^2$
 - $T(n) = 2T(n/2) + n$
- The polynomial-time mapping reduction from **Maximum Matching** (in a bipartite graph) to **Maximum Flow** in a network is most efficiently computable in _____ time with respect to the size of the bipartite graph.
 - cubic
 - quadratic
 - log-linear
 - linear
- All of the following problems have been established to be solvable in worst-case polynomial time, *except* for

- (a) finding a minimum vertex cover in a simple graph
 - (b) finding a maximum spanning tree in a weighted graph
 - (c) finding a maximum matching in a bipartite graph
 - (d) determining if an instance of **2SAT** is satisfiable
4. Based on the NP-completeness reductions provided in lecture, proving that **TSP** is NP-complete relies on each of the following reductions except for
- (a) $3\text{SAT} \leq_m^p \text{Directed Hamilton Path}$
 - (b) $3\text{SAT} \leq_m^p \text{Clique}$
 - (c) $\text{SAT} \leq_m^p 3\text{SAT}$
 - (d) $\text{Hamilton Path} \leq_m^p \text{Hamilton Cycle}$
5. In computer science, an oracle is defined as
- (a) an entity that is capable of providing answers to instances of some specified problem.
 - (b) any company that develops database software.
 - (c) an entity that is capable of providing instructions for any URM program.
 - (d) an entity that is capable of computing a universal function.
6. The mapping reduction from **Independent Set** to **Clique** is an example of a(n) ____ reduction.
- (a) embedding
 - (b) contraction
 - (c) interdomain
 - (d) none of the above