CECS 329, Homework Assignment 3, Spring 26, Dr. Ebert

Directions: Please review the `Homework` section on pages 5 and 6 of the syllabus including a list of all rules and guidelines for writing and submitting solutions.

Due Date: Saturday March 7th as a single PDF file upload to the HW3 Canvas dropbox.

Problems

1. Given `3SAT` instance

$$\mathcal{C} = \{(x_1, x_2, x_3), (\overline{x}_2, x_3, \overline{x}_4), (x_1, x_2, \overline{x}_4), (\overline{x}_1, \overline{x}_3, \overline{x}_4), (\overline{x}_1, x_2, x_4), (\overline{x}_2, x_3, x_4), (x_1, \overline{x}_3, x_4),$$
$$(\overline{x}_2, \overline{x}_3, x_4), (\overline{x}_2, \overline{x}_3, \overline{x}_4)\},$$

answer the following questions.

   a. For the mapping reduction f from `3SAT` to `Clique` presented in lecture, if $f(\mathcal{C}) = (G, k)$, then how many vertices does G have? How many edges does it have? Explain and show work. Hint: use the complement rule of counting to count the number of edges. What is the value of k? (10 pts)

   b. Does G have a k-clique? If yes, provide the vertices of the clique (for clarity make sure to indicate the vertex group of each vertex in the clique). In any case, defend your answer. (10 pts)

   c. Now suppose G is in turn mapped to the instance $(\overline{G}, k)$ of `Independent Set` using the complement reduction provided in the Mapping Reducibility lecture. How many vertices and edges does $\overline{G}$ have? Explain. (5 pts)

2. Recall the mapping reduction f from `3SAT` to `Subset Sum` presented in lecture and Suppose we apply it to the `3SAT` instance from Problem 1 to get $f(\mathcal{C}) = (S, t)$.

   a. How many members are in S? What is the largest number in S? What is the value of t? Justify your answer. (10 pts)

   b. Is there a subset A of S that sums to t? Justify your answer. In case A exists, list all of its members (hint: list each member based on its assigned name from the reduction. Do *not* provide its decimal form). Explain how you obtained this subset. (10 pts)

   c. Now suppose (S, t) is in turn mapped to the instance S' of `Set Partition` using the mapping reduction provided in the Mapping Reducibility lecture. From this reduction, we know that $S' = S \cup \{J\}$. Provide the value of J in decimal form. Show work. (10 pts)