CECS 329, Homework Assignment 5, Spring 2026, Dr. Ebert

Directions: Please review the Homework **section on pages 5 and 6 of the syllabus including a list of all rules and guidelines for writing and submitting solutions.**

Thank you to our ISA Marley Schneider for his helps with developing this HW assignment.

Due Date: Sunday, 5/3 as a single PDF file upload to the HW5 Canvas dropbox.

1 Stack Machine Programs

A Stack Machine Program is a program that can both access and manipulate a stack data structure. At the end of the program, the element at the top of the stack is the element that is returned. A program can have n inputs, where $n \geq 0$ is a nonnegative integer. These inputs are denoted as $x_0, x_1, ..., x_{n-1}$. The following is a recursive definition of what it means to be a stack.

1.1 Atomic Programs

Below is list of instructions for stack instructions for stack machine programs. A single instruction is itself a program. We call these **atomic programs** since all programs are comprised of a sequence of one or more instructions.

1. push(a): push a on to the stack
2. pop: remove the item on the top of the stack
3. add: pop the top two elements a and b from the stack (a is at the top in all these examples) and push $a + b$ on to the stack
4. sub: pop the top two elements a and b from the stack and push $a - b$ on to the stack
5. mult: pop the top two elements a and b from the stack and push $a \times b$ on to the stack
6. div: pop the top two elements a and b from the stack and push $\lfloor a/b \rfloor$ on to the stack
7. sqrt: pop a from the stack and push $\lfloor \sqrt{a} \rfloor$ on to the stack

Conventions: i) if a subtraction results in a negative number, then the result is set to 0, and ii) if there are not enough operands on the stack to meet the requirements of an arithmetic operation, or there is a division by 0, then the program will terminate with an error.

1.2 Compound Program via Concatenation

If P and Q are stack machine programs, then so is PQ, i.e. the concatenation of P followed by Q.

1.3 Example

The following is a stack machine program written in vertical form with 3 inputs. It adds x_0 and x_1 and then divides that sum by $x_2 + 7$:
push(7)
push(x_2)
add
push(x_1)
push(x_0)
add
div

The final element at the top of the stack will be equal to $(x_0 + x_1)/(x_2 + 7)$. In general, humans tend to prefer reading computer programs in vertical form, but we must remember that every program of a programming language can be expressed as a single word over some alphabet.

1.4 Word Form

We may view a stack machine program as a word over the alphabet

$$\Sigma = \{\text{push, pop, add, sub, mult, div, sqrt}, 0, 1, ..., 9, x, (,)\}$$

For example, the **word form** of the above program is

push(7) push(x_2) add push(x_1) push(x_0) add div

Note that the whitespace is only included for visual clarity. You are also welcome to use whitespace for this purpose when writing your problem solutions.

Problems

1. Use vertical form to write a stack machine program that performs the operation

   $$\lfloor\sqrt{(3x_0 + 5)/(4x_1^2 - x_2)}\rfloor.$$

   Starting (and ending) with an empty stack, show the stack contents after each instruction is processed and assuming $x_0 = 3, x_1 = 2$, and $x_3 = 9$. (10 pts).

2. Provide the word form of your program from part a. (5 pts)

3. Provide the rules for a context-free grammar that describes the set of all legal stack machine programs, where each program is viewed as a word (in word form) over Σ. Use capital letters for all variables and indicate the start variable. Conclude that the set of legal stack machine programs constitutes a context-free language. (20 pts)

4. Use your grammar to provide a leftmost derivation of the program

   push(24) push(x_3) div push(5) add

   Hint: the mathematically written x_3 stands for x3, the letter x, followed by the letter 3. (10 pts)