CECS 329, Homework Assignment 6, Spring 2026, Dr. Ebert

Directions: Solve each of the following problems. Your solution to each problem should be handwritten on a **single sheet of paper (front and back)** to be turned in before you take the exam on Thursday May 14th. Turn in **two sheets** one per problem. Plagiarized papers will result in a failing course grade. **A minimum of 40 points must be earned to qualify for passing LO11**. **Please show all work!**

Problems

1. Recall the recursive `accept` procedure described in the proof that $\texttt{Accept}_{\texttt{CFG}}$ is decidable. Draw the recursion tree that arises during the computation of `accept` on inputs G, $w = abab$, and $\nu = S_0$, where CFG G has the rules

$$S_0 \to SS \mid AB \mid WB \mid \varepsilon$$

$$S \to SS \mid AB \mid WB$$

$$W \to AS$$

$$A \to a$$

$$B \to b.$$

   Label each node of the tree with both the terminal-word and variable-word inputs that occur at that point of the recursion. For example, the root node should be labeled as $(S_0, abab)$. When substituting for the first variable of ν, follow the order of the bodies listed for that variable. For example, if the variable is S_0, then the order of substitution would be SS, AB, WB, and ε. Mark and "X" next to each leaf of the tree that represents a rejecting node and place a check next to a leaf if it is an accepting node (of course, the algorithm accepts once such a node is reached). (15 pts)

2. The goal of this problem is to establish that, given an arbitrary regular expression E over some alphabet Σ, one can decide the following property: "either E describes a language whose words all have even length, or E describes a language whose words all have odd length". The proof of decidability uses structural induction: i) decide the property for each atomic-expression input, ii) show how to decide the property for any compound expression E assuming that one can decide the property for the subexpressions that comprise E. The algorithm $\mathcal{A}$ that you will help design takes as input a single regular expression E, and outputs either 0,1, or 2, where 0 (respectively, 1) means that all words described by E have even (respectively, odd) length, while 2 implies that E has neither property.

   (a) **Atomic Step.** What should $\mathcal{A}$ output for input expression $a \in \Sigma$? for input expression ε? Explain. (5 pts)

   (b) **Compound Step (Union).** Suppose $\mathcal{A}$ decides the correct output for each of inputs E_1 and E_2. Explain how $\mathcal{A}$ decides the correct output for $E = E_1 \cup E_2$? (10 pts)

(c) **Compound Step (Concatenation).** Suppose $\mathcal{A}$ decides the correct output for each of inputs E_1 and E_2. Explain how $\mathcal{A}$ decides the correct output for $E = E_1 \circ E_2$? (20 pts) Hint: this compound step requires checking $3 \times 3 = 9$ different cases with respect to the algorithm's outputs for E_1 and E_2.

(d) **Compound Step (Star).** Suppose $\mathcal{A}$ decides the correct output for E_1. Explain how $\mathcal{A}$ decides the correct output for $E = E_1^*$? (10 pts)

Hint: in all compound cases we may assume that $E_1 \neq \emptyset$ and $E_2 \neq \emptyset$.