CECS 329, Exam 1, Spring 2026, Dr. Ebert

IMPORTANT: READ THE FOLLOWING DIRECTIONS SO YOU WILL NOT LOSE POINTS. Directions: This exam has SIX different problems: one problem for each of LO's 1-3 and three additional problems.

- For each problem, write your solution using **ONE SHEET OF PAPER ONLY (BOTH FRONT AND BACK)**. Write **NAME** and **PROBLEM NUMBER** on each sheet.
- Write solutions to different problems on **SEPARATE SHEETS** of paper.
- For example, if you decide to solve all six problems, then you will submit **SIX** sheets for grading.

Unit 1 LO Problems (25 pts each)

LO1. Do the following.

(a) Consider the function `sol` which accepts two inputs: a subset of natural numbers S and a nonnegative integer t, and returns the unique subset A of S whose members sum to t. Evaluate $\texttt{sol}(\{3, 6, 8, 9, 15\}, 27)$. Also, is it true that

$$\{\{3\}, \{2\}\} \in \{\{4\}, \{2\}, \{3\}, \{1\}\}?$$

Explain. (10 pts)

(b) Consider the `2SAT` instance

$$\mathcal{C} = \{(x_1, \overline{x}_4), (\overline{x}_1, x_2), (x_2, \overline{x}_3), (x_2, x_5), (\overline{x}_3, x_5), (\overline{x}_3, x_6), (x_4, \overline{x}_5), (x_4, x_6), (x_5, \overline{x}_6)\}.$$

i. Draw the implication graph $G_{\mathcal{C}}$. (5 pts)

ii. Perform the `Improved 2SAT` algorithm by computing the necessary reachability sets. Use numerical order (in terms of the variable index) and positive literal before negative literal when choosing the reachability set to compute next. Draw the resulting reduced `2SAT` instance whenever a consistent reachability set is computed. Either provide a final satisfying assignment for $\mathcal{C}$ or indicate why $\mathcal{C}$ is unsatisfiable. (10 pts)

LO2. Do the following.

(a) Provide the general definition of what it means to be a mapping reduction from problem A to problem B. Hint: do *not* assume that A and B are decision problems. (6 pts)

(b) Recall the mapping reduction $f : \texttt{SP} \rightarrow \texttt{SS}$ from `Set Partition` to `Subset Sum` provided in lecture. Compute $f(S)$ for

$$S = \{2, 5, 7, 8, 9, 12, 16, 23\}.$$

(7 pts)

(c) Verify that f is a valid mapping reduction for input S in the sense that S and $f(S)$ are either both positive instances or both negative instances of their respective decision problems. **Defend your answer**. (12 pts)

LO3. Answer the following. Note: a minimum total of 16 points must be scored in order to pass LO3.

(a) An instance of the `Quadratic Diophantine (QD)` decision problem is a triple (a, b, c) of positive integers, and the problem is to decide there are nonnegative integers x and y for which

$$ax^2 + by = c.$$

Which of the following best describes the certificate input for a `QD` verifier that establishes $\texttt{QD} \in \texttt{NP}$? (6 pts)

i. a triple of positive integers (a, b, c)

ii. the quadratic equation $ax^2 + by = c$

iii. a pair of nonnegative integers (x, y)

iv. a triple of positive integers (a, b, c) *and* a pair of nonnegative integers (x, y)

(b) An instance of the `Boolean Vector Sum (BVS)` decision problem is a pair (S, k), where S is a set of Boolean vectors, each having the same length, and $k \geq 1$ is a natural number. The problem is to decide if there are k different vectors $v_1, v_2, \ldots, v_k \in S$ for which

$$v_1 + v_2 + \cdots + v_k = (1\ 1\ \cdots 1),$$

where + represents bitwise (inclusive) OR. For example,

$$S = \{(1\ 0\ 1\ 0\ 0), (0\ 0\ 1\ 0\ 1), (0\ 1\ 0\ 0\ 1), (0\ 1\ 0\ 1\ 1), (1\ 0\ 0\ 1\ 0)\}$$

and $k = 3$ is a positive instance of `BVS` since

$$(1\ 0\ 1\ 0\ 0) + (0\ 1\ 0\ 0\ 1) + (1\ 0\ 0\ 1\ 0) = (1\ 1\ 1\ 1\ 1).$$

Provide size parameters for the `BVS` decision problem. Clearly define each parameter. Hint: there are two of them. (6 pts)

(c) Classify each of the following problems as being in `P`, `NP`, or co-`NP` (3 points each).

i. An instance of decision problem `Zero Matrix Power (ZMP)` consists of an $n \times n$ matrix A having integer entries and the problem is to decide if $A^n = 0$.

ii. An instance of decision problem `Fallacy` is a Boolean formula $F(x_1, \ldots, x_n)$, and the problem is to decide if there is no assignment α over the variables of F that can make F evaluate to 1.

iii. An instance of decision problem `VC37` is a simple graph $G = (V, E)$ and the problem is to decide if there exists a vertex cover for G with size equal to 37.

iv. The `Hamilton Path` decision problem presented in the Mapping Reducibility lecture.

Additional Problems (25 pts each)

A1. Let $\mathcal{C}$ be a satisfiable instance of `2SAT`, and let α be a satisfying assignment for $\mathcal{C}$. Answer the following in relation to $\mathcal{C}$.

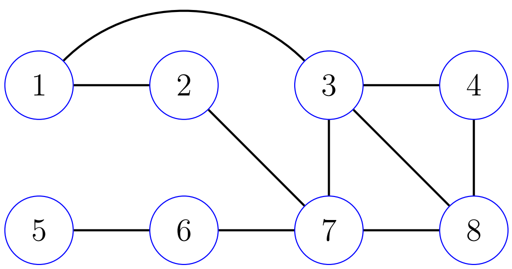
(a) If $P = \overline{x}_3, x_6, \overline{x}_1, \overline{x}_4, x_3$ is a path in the implication graph $G_{\mathcal{C}}$, then what can you say about satisfying assignment α? (7 pts)

(b) Provide another length-4 path that must also appear in $G_{\mathcal{C}}$. Explain. (6 pts)

(c) List all clauses that are surely in $\mathcal{C}$. Explain. (6 pts)

(d) Provide two clauses that, if they were added to $\mathcal{C}$, would make $\mathcal{C}$ unsatisfiable. Explain. (6 pts)

A2. The simple graph $G = (V, E)$ shown below and $k = 4$ together make the instance $(G, k = 3)$ of the `Clique` decision problem.



(a) Provide $f(G, k)$, where f is the mapping reduction from `Clique` to `Half Clique` provided in the core exercises of the Mapping Reducibility lecture. (10 pts) Note: correctly solving this problem counts for passing one half of LO2.

(b) Show that f is a valid mapping reduction for input (G, k) in the sense that (G, k) and $f(G, k)$ are either both positive instances or both negative instances of their respective decision problems. **Defend your answer**. (15 pts)

A3. Recall the `Vertex Cover (VC)` decision problem that was defined in the Mapping Reducibility lecture. Let $G = (V, E)$ and $k \geq 0$ be an instance of `VC`. Let C be a subset of k vertices that serves as a certificate for (G, k).

(a) Provide pseudocode for a verifier program that takes as inputs both (G, k) and C, and returns 1 iff C is a vertex cover of size k for G. (15 pts)

(b) Provide size parameters for `VC` and use them to describe the number of steps required by your verifier from part a. Defend you answer. (10 pts)