# CECS 329, Exam 2 Spring 2026, Dr. Ebert

**IMPORTANT: READ THE FOLLOWING DIRECTIONS.**

- For each of LO's 4-7 and the Additional problems, write your solution using a **SINGLE** and **SEPARATE SHEET OF PAPER ONLY (BOTH FRONT AND BACK)**. Write **NAME** and **PROBLEM NUMBER** on each sheet.
- For makeup LO's it's OK to use the same sheet for two or more problems if there is sufficient space.

## Unit-2 LO's (25 Points Each)

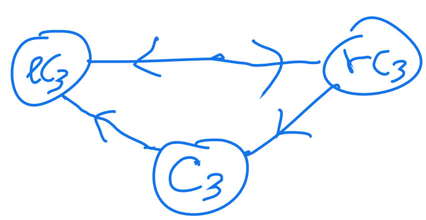
LO4. Do the following.

a. Consider the `3SAT` instance

$$\mathcal{C} = \{c_1 = (x_1, x_2, \overline{x}_4), c_2 = (\overline{x}_2, \overline{x}_3, x_4), c_3 = (\overline{x}_1, x_2, x_3), c_4 = (x_1, \overline{x}_3, \overline{x}_4)\}.$$

Consider the mapping reduction $f$ : `3SAT` $\rightarrow$ `DHP` from `3SAT` to `Directed Hamilton Path` that was presented in lecture, where $f(\mathcal{C}) = (G, a, b)$. Note: to pass part a of LO4, two of the following three parts must be correctly answered.

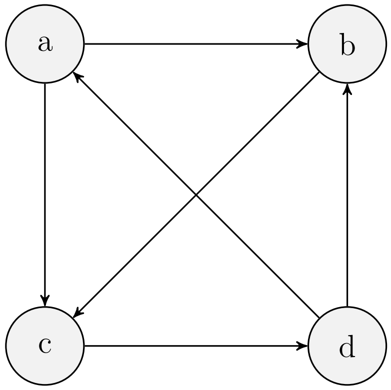
i. Consider the vertices $\mathrm{lc}_3$ and $\mathrm{rc}_3$ located in the $x_2$-diamond. Draw the three vertices $\mathrm{lc}_3$, $\mathrm{rc}_3$, and $c_3$ as well as the edges that exist between these vertices. (5 pts)
**Solution.**




ii. Which diamond has no edge connections to or from clause vertex $c_4$ and why? (5 pts)
**Solution.** The $x_2$ diamond does connect to vertex $c_4$ since variable $x_2$ does not appear in clause $c_4$ and thus a path should not be able to visit (i.e. satisfy) clause $c_4$ via $x_2$.

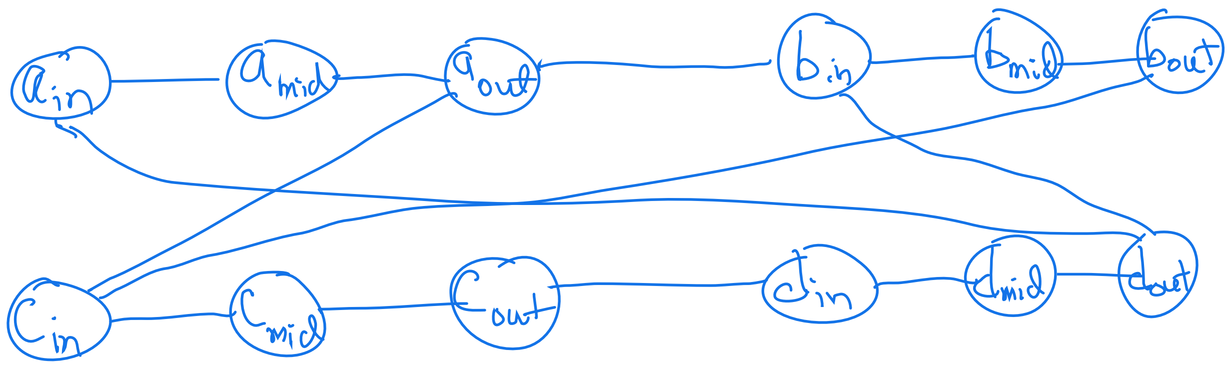
iii. Consider the assignment $\alpha = (x_1 = 1, x_2 = 0, x_3 = 1, x_4 = 0)$ that satisfies $\mathcal{C}$ and use it to describe a **trip itinerary** from vertex $a$ to vertex $b$, where, for each variable diamond, the itinerary describes the direction of movement (left to right or right to left) through the diamond, as well as the clause vertices that are vistited while moving through the diamond. (5 pts)
**Solution.** Move right to left through diamond $x_1$ and visit $c_1$ and $c_4$ along the way. Move left to right through diamond $x_2$ and visit $c_2$ along the way. Move right to left through diamond $x_3$ and visit $c_3$ along the way. Move left to right through diamond $x_4$ and do not visit any clauses along the way.

b. Given the graph $G$ shown below, provide $f(G, a, b)$, where $f$ is the mapping reduction from DHP to UHP. In otherwords, draw the graph $G'$ and provide both the start and end vertices for the desired Hamilton path in $G'$. (10 pts)




**Solution.**

$G'$:

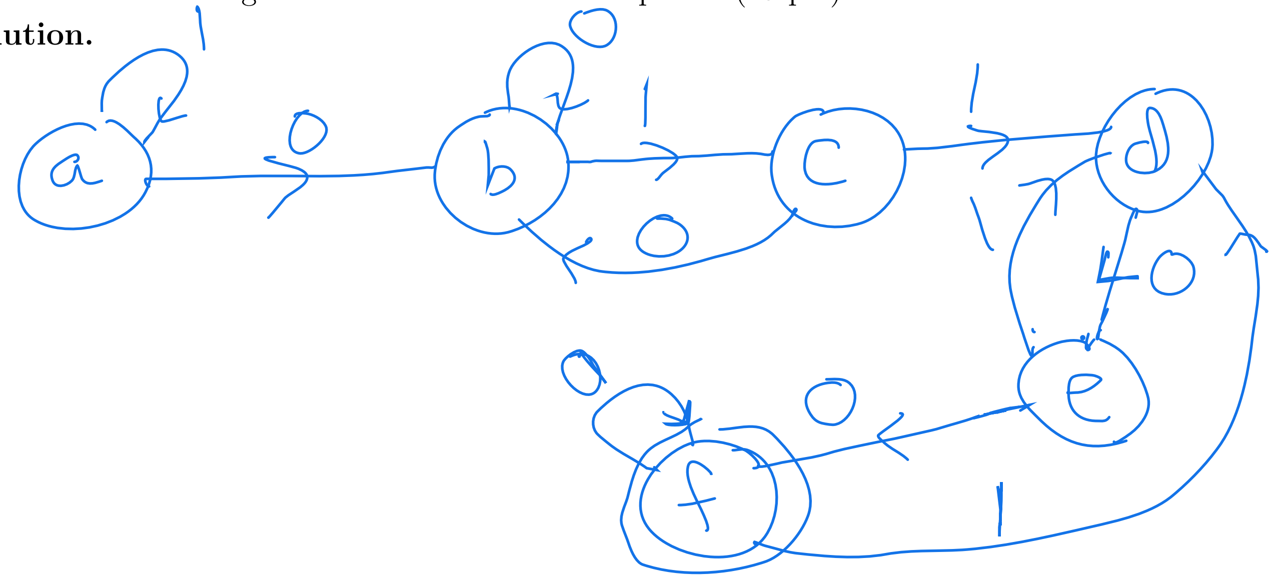



$a' = a_{in}$ $b' = b_{out}$

LO5. Do the following.

a. Let $L$ be the language of binary words that contain the subword 011 and end with a 00. Provide the state diagram for a DFA $M$ that accepts $L$. (19 pts)

**Solution.**




b. Demonstrate the computation of $M$ on inputs i) $w_1 = 0110100$ and ii) $w_2 = 010100$. For each computation, indicate whether $w$ is accepted or rejected. (6 pts)

**Solution.**

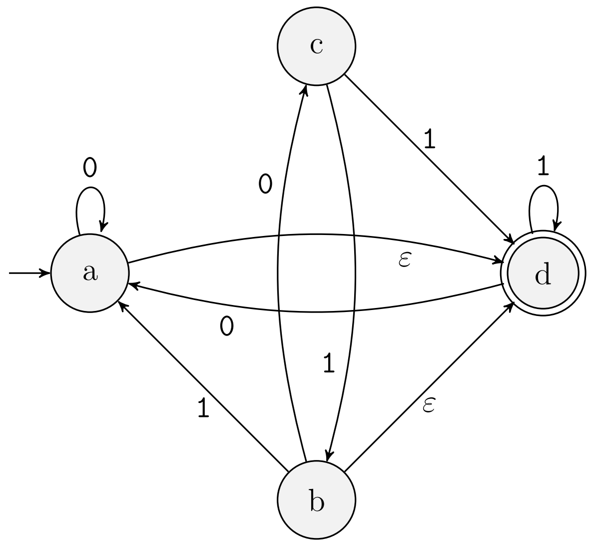
$w_1 =$ 0 1 1 0 1 0 0

a b c d e d e f ⇒ accept

$w_2 =$ 0 1 0 1 0 0

a b c b c b b ⇒ reject

LO6. Do the following for the NFA $N$ whose state diagram is shown below.



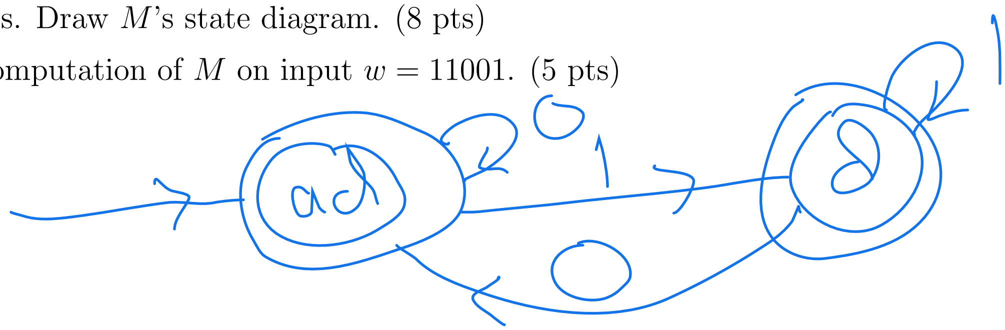

| | 0 | 1 |
|---|---|---|
| a | ad | ∅ |
| b | c | ad |
| c | ∅ | bc |
| d | ad | d |

a. Provide a table that represents $N$'s $\delta$ transition function. (12 pts)

b. Use the table from part a to convert $N$ to an equivalent DFA $M$ using the method of subset states. Draw $M$'s state diagram. (8 pts)

c. Show the computation of $M$ on input $w = 11001$. (5 pts)

**Solution.**




w = 1 1 0 0 1

ad d d ad ad d

⇓

accept

LO7. Do the following.

a. Let $L_1$ denote the language of binary words that contain exactly two 0's and exactly two 1's. Let $L_2$ denote the language consisting of all binary words $w$ having length four and that (when viewed as a binary number) is divisible by 3. Use set notation to write the members of $L_1 \cup L_2$. (6 pts)

**Solution.** $L_1 \cup L_2 = \{1100, 1010, 1001, 0110, 0101, 0011, 0000, 1111\}$

b. Let $L$ denote the language of binary words that contain exactly two 1's and an even number of 0's. Is 11000101010010 a member of $L^*$? Justify your answer. (6 pts)

**Solution.** Yes, since $1100, 0101, 010010 \in L$ and the concatentation of these three words equals 11000101010010.

c. Provide a regular expression that describes the language consisting of all binary words that have an even number of 0's and for which every odd bit is 1. Examples of words that are in this language include $\varepsilon$, 1111, 10101, 101110111010. (13 pts)

**Solution.** $(11)^*((11)^*10(11)^*10)^*(11)^*$ $\{1, \varepsilon\}$

## Additional Problems

A1. Answer the following.

(a) Provide the definition of what it means for a decision problem to be `NP`-complete. (6 pts)

**Solution.** See lecture notes.

(b) Professor Scharlemann has just discovered a polynomial-step algorithm $\mathcal{A}$ for solving instances of the `Clique` decision problem. The algorithm requires $O(m^5 n^3)$ steps, where $m = |E|$ and $n = |V|$ are the size parameters of the input graph. Why does his algorithm also yield a polynomial-step algorithm $\mathcal{A}'$ for solving the `3SAT` decision problem? Provide the big-O number of steps that such an algorithm would require to solve an instance of `3SAT` that has $m$ clauses. (10 pts)

**Solution.** First map `3SAT` instance $\mathcal{C}$ to `Clique` instance $G$, $k = m$, via the algorithm provided in lecture. This requires $O(m + m^2) = O(m^2)$ steps. Also the resulting graph $G$ has $O(m)$ vertices and $O(m^2)$ edges. Thus, when applying Scharlemann's algorithm to $G$, the number of steps required is $O((m^2)^5 m^3) = O(m^{13})$ and so the total number of steps for the entire process equals $O(m^2 + m^{13}) = O(m^{13})$, which is a polynomial upper bound. Finally, notice that, since the map used is a mapping reduction, the answer returned by Scharlemann's algorithm will equal the answer to the original problem instance $\mathcal{C}$.

(c) A **nondeterministic finite automaton (NFA)** $N$ consists of a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

$$\delta : Q \times \Sigma \to \mathcal{P}(Q),$$

maps a each (state, input) pair to a subset of next states. When converting $N$ to a DFA $M$ using the method of subset states, i) describe $M$'s set of states, ii) what is $M$'s initial state? and iii) what constitutes a final state for $M$? (9 pts)

**Solution.** See lecture notes.

A2. Answer the following.

(a) The **reversal** of a word $w$, written $w^r$, is $w$ but written in reverse order. For example $\text{sleep}^r = \text{peels}$. Similarly, if $L$ is a language over some alphabet $\Sigma$, then $L^r$ denotes the language consisting of all the words in $L$, with each one written in reverse order. For example, $\{01, 1011, 0011\}^r = \{10, 1101, 1100\}$. With your help we now show that if $L$ is regular, then so is $L^r$.

i. Compute $\{\varepsilon\}^r$, $\emptyset^r$, and $\{a\}^r$, where $a \in \Sigma$. (3 pts)
**Solution.** $\{\varepsilon\}^r = \{\varepsilon\}$, $\emptyset^r = \emptyset$, $\{a\}^r = \{a\}$

ii. Suppose $L_1$ and $L_2$ are both regular languages. Provide a formula for computing $(L_1 \cup L_2)^r$ in terms of $L_1^r$ and $L_2^r$. (3 pts)
**Solution.** $(L_1 \cup L_2)^r = L_1^r \cup L_2^r$

iii. Suppose $L_1$ and $L_2$ are both regular languages. Provide a formula for computing $(L_1 \circ L_2)^r$ in terms of $L_1^r$ and $L_2^r$. (3 pts)
**Solution.** $(L_1 \circ L_2)^r = L_2^r \circ L_1^r$.

iv. Suppose $L$ is a regular language. Provide a formula for computing $(L^*)^r$ in terms of $L^r$. (3 pts)
**Solution.** $(L^*)^r = (L^r)^*$

(b) Consider the alphabet

$$\Sigma = \{\begin{pmatrix} i \\ j \end{pmatrix} \mid i, j \in \{0, 1, \ldots, 9\}$$

consisting of binary vectors whose entries are the digits $0, 1, \ldots, 9$. Moreover, consider the language $L \subseteq \Sigma^*$ consisting of all words for which the bottom layer represents a decimal number that is three times the value of the decimal number in the top layer, where both numbers are written, from left to right, starting with the least significant digit (LSD) to the most significant digit (MSD). For example, the word

$$\begin{pmatrix} 9 \\ 7 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

is a positive instance of $L$ since the bottom number 27 is three times the top number 9. We may decide $L$ using a DFA whose set of states are carry 0, carry 1, carry 2, and reject, where the initial and final states are both carry 0. Moreover, if the current state is carry $c$, where $c \in \{0, 1, 2\}$ and the current vector is

$$\begin{pmatrix} i \\ j \end{pmatrix},$$

e.g. $C_0$: "carry 0"

then to avoid the reject state, it must be the true that $j = 3i + c \mod 10$. In this case, the next state will be carry $c'$, where $c' = \lfloor \frac{3i+c}{10} \rfloor$. List all the vectors that will *not* be rejected when in state carry 2. For each such vector, provide the associated next state. (13 pts)

**Solution.**

$\begin{pmatrix} 0 \\ 2 \end{pmatrix} \Rightarrow C_0$  $\begin{pmatrix} 1 \\ 5 \end{pmatrix} \Rightarrow C_0$  $\begin{pmatrix} 2 \\ 8 \end{pmatrix} \Rightarrow C_0$

$\begin{pmatrix} 3 \\ 1 \end{pmatrix} \Rightarrow C_1$  $\begin{pmatrix} 4 \\ 4 \end{pmatrix} \Rightarrow C_1$  $\begin{pmatrix} 5 \\ 7 \end{pmatrix} \Rightarrow C_1$  $\begin{pmatrix} 6 \\ 0 \end{pmatrix} \Rightarrow C_2$

# Unit-1 LO's (0 Points Each)

$\binom{7}{3} \Rightarrow C_2$ $\binom{8}{6} \Rightarrow C_2$ $\binom{9}{9} \Rightarrow C_2$

LO1. Do the following.

(a) Is it true that $\{\{1\}, \{2, 3\}, \{4, 5, 6\}\} \subseteq \{1, 2, 3, 4, 5\}$? **Explain**. Also, if function

$$f : \mathcal{N} \to \mathcal{P}(\mathcal{N})$$

is defined by $f(n)$ equals the set of all natural numbers that divide evenly into $n$, then compute $f(23)$ and $f(42)$. Hint: non-primes should also be included.

(b) Consider the `2SAT` instance

$$\mathcal{C} = \{(x_1, \overline{x}_2), (x_1, \overline{x}_5), (\overline{x}_1, x_3), (\overline{x}_1, \overline{x}_6), (\overline{x}_2, x_5), (x_3, \overline{x}_4), (\overline{x}_3, \overline{x}_4), (x_4, x_6), (x_4, \overline{x}_6)\}.$$

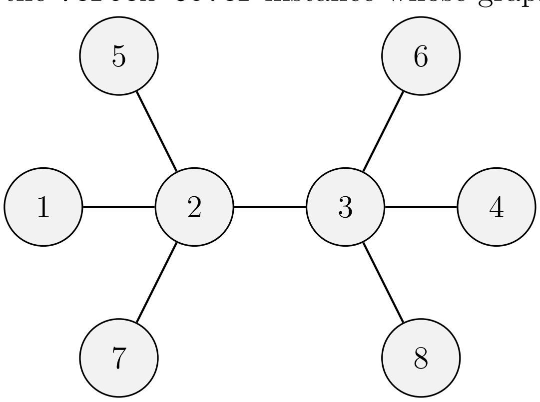
i. Draw the implication graph $G_{\mathcal{C}}$.

ii. Perform the `Improved 2SAT` algorithm by computing the necessary reachability sets. Use numerical order (in terms of the variable index) and positive literal before negative literal when choosing the reachability set to compute next. Draw the resulting reduced `2SAT` instance whenever a consistent reachability set is computed. Either provide a final satisfying assignment for $\mathcal{C}$ or indicate why $\mathcal{C}$ is unsatisfiable.

LO2. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from decision problem $A$ to decision problem $B$.

(b) For the mapping reduction $f$ : `Vertex Cover` $\to$ `Half Vertex Cover`, draw $f(G, k)$ for the `Vertex Cover` instance whose graph is shown below, and for which $k = 2$.




(c) Verify that both $(G, k)$ and $f(G, k)$ are either both positive instances, or are both negative ones. Explain and show work.

LO3. Answer the following. Note: a minimum total of 15 points must be scored in order to pass.

(a) An instance of `Feedback Arc Set (FAS)` is a directed graph $G = (V, E)$ and a natural number $k \geq 0$. The problem is to decide if there is a set $S$ of $k$ vertices of $G$ for which, when removing the vertices of $S$ from $G$ (and all edges incident with them) the resulting graph is acyclic. Which of the following best describes the certificate input for a `FAS` verifier that establishes `FAS` $\in$ `NP`? (6 pts)

i. a set of edges from $E$

ii. $G = (V, E)$ and a set of edges from $E$

iii. a set of vertices from $V$

iv. $G = (V, E)$ and a set of vertices from $V$

**Solution.** iii, but partial credit for iv

(b) Provide size parameters for `FAS`. (6 pts)

**Solution.** $m = |E|$, $n = |V|$.

(c) Classify each of the following problems as being in `P`, `NP`, or co-`NP` (3 points each).

i. An instance of the `Substring` decision problem is a pair of binary strings $(s_1, s_2)$, and the problem is to decide if $s_1$ is a substring of $s_2$.

ii. An instance of `Spanning Tree` is a weighted graph $G = (V, E, w)$ and a nonnegative integer $k$ and the problem is to decide if $G$ has a spanning tree whose edge weights sum to a value that does not exceed $k$.

iii. An instance of the `Fallible` decision problem is a Boolean formula $F$ and the problem is to decide if there is an assignment $\alpha$ to the variables of $F$ for which $F(\alpha) = 0$.

iv. An instance of `Cubic Diophantine` is a triple $(a, b, c, d)$ of natural numbers and the problem is to decide if there does not exist positive integers $x, y, z > 0$ for which

$$ax^3 + bx^2 + cx = d.$$

Hint: this problem is not in P.

**Solution.** P,P,NP,co-NP