

LO1:

$$a) \{ \{1\}, \{3, 4\}, \{1, 3, 5\} \} \notin P(\{1, 2, 3, 4, 5\})$$

The powerset of a subset contains all possible subsets of a set, so while $P(\{1, 2, 3, 4, 5\})$ would contain $\{1\}$, $\{3, 4\}$, and $\{1, 3, 5\}$, it would not contain the set of those sets.

$$f(17) = \{1, 17\}$$

$$f(68) = \{1, 2, 4, 17, 34, 68\}$$

$$b) i) (x_1, x_2): \bar{x}_1 \rightarrow x_2, \bar{x}_2 \rightarrow x_1$$

$$(x_1, x_6): \bar{x}_1 \rightarrow x_6, \bar{x}_6 \rightarrow x_1$$

$$(\bar{x}_1, \bar{x}_2): x_1 \rightarrow \bar{x}_2, x_2 \rightarrow \bar{x}_1$$

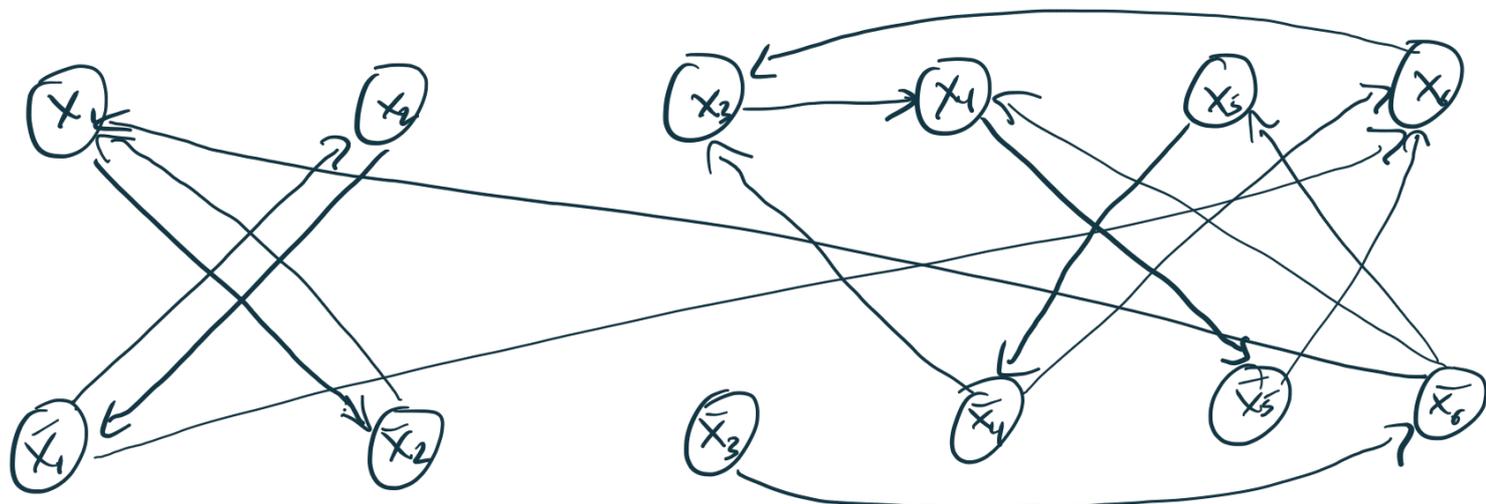
$$(x_3, \bar{x}_6): \bar{x}_3 \rightarrow \bar{x}_6, x_6 \rightarrow x_3$$

$$(\bar{x}_3, x_4): x_3 \rightarrow x_4, \bar{x}_4 \rightarrow x_3$$

$$(\bar{x}_4, \bar{x}_5): x_4 \rightarrow \bar{x}_5, x_5 \rightarrow \bar{x}_4$$

$$(x_4, x_6): \bar{x}_4 \rightarrow x_6, \bar{x}_6 \rightarrow x_4$$

$$(x_5, x_6): \bar{x}_5 \rightarrow x_6, \bar{x}_6 \rightarrow x_5$$

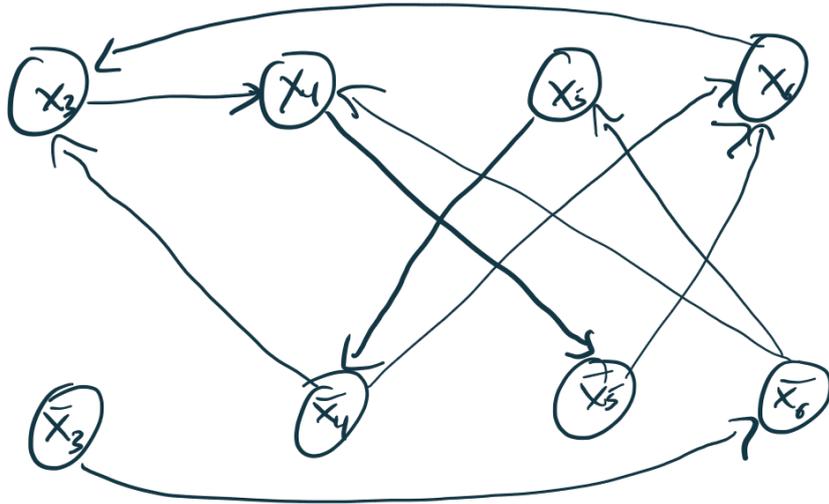


ii) $R_{x_1} = \{x_1, \bar{x}_2\}$ this is consistent, so we assign and reduce the graph

$$x_1 = 1$$

$$x_2 = 0$$

Reduced Graph:



$R_{x_3} = \{x_3, x_4, \bar{x}_5, x_6\}$ this is consistent, so we assign, and get our final satisfying assignment.

$$x_3 = x_4 = x_6 = 1$$

$$x_5 = 0$$

$$d = \{1, 0, 1, 1, 0, 1\}$$

L02:

a) A mapping reduction from Problem A to Problem B means for any positive instance of problem A, it will turn it into a positive instance of Problem B; any negative instance of problem A will be turned into a negative instance of Problem B.

b) $\text{sum}(S) = 36$
 $t = 36/2 = 18$ \rightarrow Subset Sum Instance:
 $t = 18$ $S = \{1, 3, 6, 10, 16\}$

c) f makes it so both problems are asking the same question by setting t to $\text{sum}(S)/2$. If we can find a subset equal to t , we will always be able to split the set into two parts with equal sum, making both instances positive. If we cannot find a subset equal to t , we won't be able to split the set, making both instances negative. Therefore, f is a valid mapping reduction.