CECS 329, Quiz 2 Solutions, Spring 2026, Dr. Ebert

IMPORTANT: READ THE FOLLOWING DIRECTIONS.

- For each problem, write your solution using **ONE SHEET OF PAPER ONLY (BOTH FRONT AND BACK)**. Write **NAME** and **PROBLEM NUMBER** on each sheet.
- Write solutions to different problems on **SEPARATE SHEETS** of paper.
- For Makeup problems you may either solve **ONE WHOLE PROBLEM** OR **HALF OF TWO DIFFERENT PROBLEMS**. Any additional solutions will **NOT** be graded.

Unit 2 LO Problems

LO4. Answer the following.

(a) Consider the `3SAT` instance

$$\mathcal{C} = \{c_1 = (x_1, x_2, \overline{x}_3), c_2 = (\overline{x}_1, \overline{x}_2, \overline{x}_3), c_3 = (\overline{x}_1, x_2, x_3), c_4 = (x_1, \overline{x}_2, \overline{x}_3)\}.$$

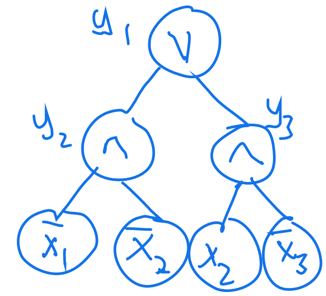
and the mapping reduction f : `3SAT` $\to$ `Subset Sum` from `3SAT` to `Subset Sum` that was presented in lecture. Do the following: i) draw the complete table associated with $f(\mathcal{C}) = (S, t)$, including target t, and ii) provide a subset A of S that is a direct consequence of $\alpha = (x_1 = 1, x_2 = 0, x_3 = 1)$ being an assignment that satisfies $\mathcal{C}$ and verify that the members of A sum to t. When listing the members of A, use their corresponding variable names (i.e. y's, z's, g's, and h's).

				c1	c2	c3	c4
y_1	1	0	0	1	0	0	1
z_1	1	0	0	0	1	1	0
y_2		1	0	1	0	1	0
z_2		1	0	0	1	0	1
y_3			1	0	0	1	0
z_3			1	1	1	0	1
g_1				1	0	0	0
h_1				1	0	0	0
g_2					1	0	0
h_2					1	0	0
g_3						1	0
h_3						1	0
g_4						0	1
h_4						0	1
t	1	1	1	3	3	3	3

$A = \{y_1, z_2, y_3, g_1, h_1, g_2, h_2, g_3, h_3, g_4\}$

(b) Answer the following regarding the mapping reduction from SAT to 3SAT that uses the Tseytin transformation.

i. If the Boolean formula $F(x_1, x_2, x_3) = (\overline{x}_1 \wedge \overline{x}_2) \vee (x_2 \wedge \overline{x}_3)$, then provide the initial Boolean formula that is satisfiability equivalent to F. Hint: one of the subformulas is $y_3 \leftrightarrow (x_2 \wedge \overline{x}_3)$.



$y_1 \wedge (y_1 \leftrightarrow (y_2 \vee y_3)) \wedge (y_2 \leftrightarrow (\overline{x}_1 \wedge \overline{x}_2))$
$\wedge (y_3 \leftrightarrow (x_2 \wedge \overline{x}_3))$

ii. By using the steps of the Tseytin transformation described in lecture, show how to convert $y_3 \leftrightarrow (x_2 \wedge \overline{x}_3)$ to a set of 3SAT clauses.

$\Updownarrow$

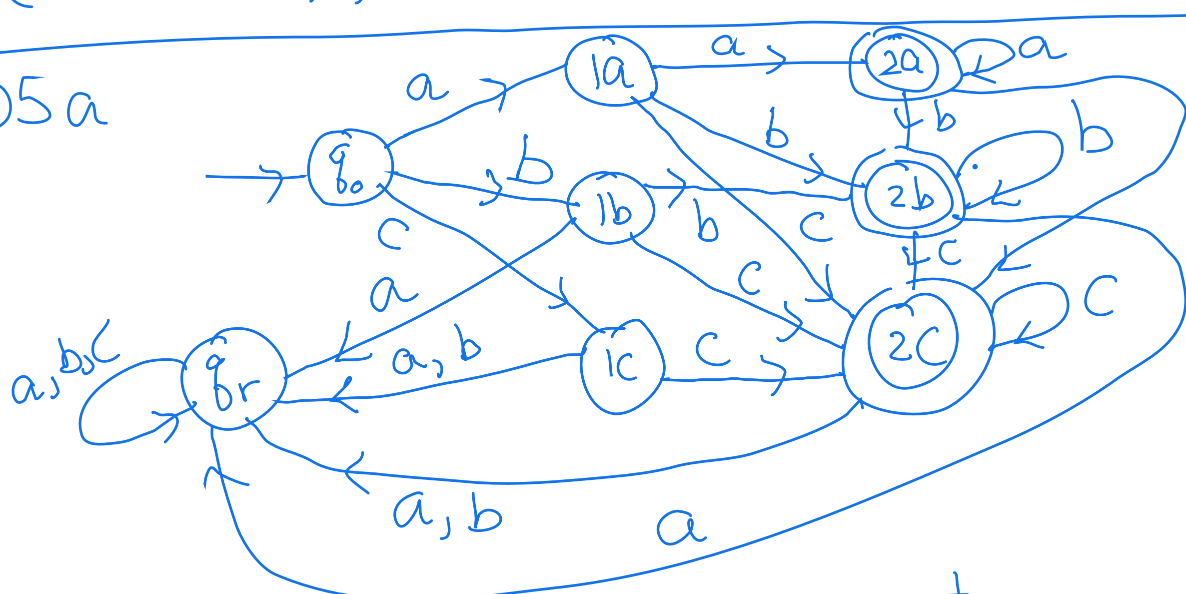
$(y_3 \rightarrow (x_2 \wedge \overline{x}_3)) \wedge ((x_2 \wedge \overline{x}_3) \rightarrow y_3) \iff$

$(\overline{y}_3 \vee (x_2 \wedge \overline{x}_3)) \wedge (\overline{x_2 \wedge \overline{x}_3} \vee y_3) \iff$

$(\overline{y}_3 \vee x_2) \wedge (\overline{y}_3 \vee \overline{x}_3) \wedge (\overline{x}_2 \vee x_3 \vee y_3) \iff$

$\{(\overline{y}_3, x_2, x_2), (\overline{y}_3, \overline{x}_3, \overline{x}_3), (\overline{x}_2, x_3, y_3)\}$

i LO5a



b) a a b c
q_0 1a 2a 2b 2c → accept

a b c a b

q_0 1a 1b 1C q_r q_r $\Rightarrow$ reject

LO5. Do the following.

(a) Consider the language L of all words w over the alphabet {a,b,c} for which i) $|w| \geq 2$, and ii) w is *sorted*, meaning that it begins with zero or more a's, followed by zero or more b's, and ending with zero or more c's. For example aa and abbc are members of L, but c and bac are not members of L.

(b) Demonstrate the computation of M on inputs i) $w_1 =$ aabc and ii) $w_2 =$ abcab. For each computation, indicate whether w is accepted or rejected.

Makeup Problems

LO1. Solve the following.

(a) Consider function $\texttt{sum} : \mathcal{P}(\mathcal{N}) \to \mathcal{N}$ whose domain is sets of natural numbers and whose codomain is the set of natural numbers. Indeed, when a set S is input into `sum`, then the output is the sum of all the members of S. Use function notation to evaluate `sum` on input $\{3, 5, 9, 14\}$. Credit will not be awarded if function notation is not properly used. Also, If $\mathcal{S}$ is a set whose six members are also sets, each having size two, and each a subset of $\{1, 2, 3, 4\}$ then use list notation to describe $\mathcal{S}$.

(b) Consider the `2SAT` instance

$$\mathcal{C} = \{(x_1, \overline{x}_2), (\overline{x}_1, \overline{x}_3), (\overline{x}_1, \overline{x}_4), (\overline{x}_1, x_6), (x_2, x_4), (x_3, \overline{x}_6), (\overline{x}_3, x_5), (\overline{x}_5, x_6), (\overline{x}_5, \overline{x}_6)\}.$$

i. Draw the implication graph $G_{\mathcal{C}}$.

ii. Perform the `Improved 2SAT` algorithm by computing the necessary reachability sets. Use numerical order (in terms of the variable index) and positive literal before negative literal when choosing the reachability set to compute next. Draw the resulting reduced `2SAT` instance whenever a consistent reachability set is computed. Either provide a final satisfying assignment for $\mathcal{C}$ or indicate why $\mathcal{C}$ is unsatisfiable.

LO2. Answer the following.

(a) Provide the definition of what it means to be a mapping reduction from problem A to problem B.

(b) Recall the mapping reduction $f : \texttt{SP} \to \texttt{SS}$ from `Set Partition` to `Subset Sum` provided in lecture. Compute $f(S)$ for

$$S = \{2, 4, 7, 11, 18\}.$$

(c) Verify that f is a valid mapping reduction for input S in the sense that S and $f(S)$ are either both positive instances or both negative instances of their respective decision problems. **Defend your answer**.

LO3. Answer the following. Note: a minimum total of 16 points must be scored in order to pass.

(a) Recall the `Hamilton Path (HP)` decision problem for which an instance is a simple graph $G = (V, E)$ and two vertices $a, b \in V$. Which of the following best describes the certificate input for a `HP` verifier that establishes $\texttt{HP} \in \texttt{NP}$? (6 pts)

i. a sequence of vertices from V
ii. a set of vertices from V
iii. a subgraph H of G
iv. a set of edges from E

(b) An instance of the `Quadratic Diophantine (QD)` decision problem is a triple (a, b, c) of positive integers, and the problem is to decide there are nonnegative integers x and y for which

$$ax^2 + by = c.$$

Provide a *single* size parameter for the `QD` decision problem. (6 pts)

(c) Classify each of the following problems as being in `P`, `NP`, or co-`NP` (3 points each).

i. An instance of the `MineSweep` decision problem is a simple graph $G = (V, E)$ for which some of G's nodes are labeled with a positive integer. The problem is to decide if there is a way to place mines on some of G's unlabeled nodes so that, for each $v \in V$ that is labeled with some integer k, exactly k of v's neighbors have been assigned a mine.

ii. An instance of the `Connected` decision problem is a simple graph $G = (V, E)$. The problem is to decide if G is connected, meaning that there is a path between any two vertices in G.

iii. An instance of the `Maximum Cut` decision problem is a graph $G = (V, E)$ and an integer $k \geq 0$. The problem is to decide if there is a way of coloring the vertices of G using the colors red and blue, so that there are at least k edges of the form (u, v) for which u and v have been assigned different colors.

iv. An instance of the `Small Cliques` decision problem is a simple graph $G = (V, E)$ and the problem is to decide if all cliques of G have a size that is at most one percent of the total number of vertices.